MST based Pyramid Model of TSP

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Multiresolution Pyramid Representation

Approach

- Local to global: bottom-up reduction of resolution
- Global to local: top-down solution refinement

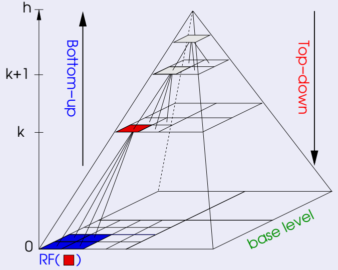
Approximate the TSP solution using pyramid representation
The idea

1. partition the input space
   - preserve approximate location
2. reduce number of cities
3. repeat until solution becomes trivial
4. refine solution top down to the base level

Image Pyramids
Introduction

- Hierarchical structures in computer vision
  ⇒ image pyramids, wavelets, quad-trees ...
- Characteristics of pyramids:
  - Structure
    - horizontal and vertical relations
  - Content of the cells
    - numeric, symbolic or both
  - Processing of a cells

$2 \times 2/4$ Regular pyramid



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Outline of the Talk

1. Image Pyramids
2. Minimum Spanning Tree
3. Approximate Traveling Salesman Problem
4. Summary

How to Organize/Partition City-Space?

1. Raster cell with/without city
2. Graph $G = (V, E)$: city = vertex $v \in V$; edges $e \in E$?
   - Complete graph: $E = V \times V$
   - Delaunay triangulation $E \subset V \times V$ and Voronoi Diagram

Traveling Salesman Problem (TSP)

- Fully connected graph $G = (V, E, w)$ and attributed by weight w
  - costs $w : e \in E \rightarrow R^+$
- Goal : Find the tour τ with the smallest weight $\sum_{e \in \tau} w(e) \rightarrow \min$.
  - If the weights are (2D) Euclidean distances $\rightarrow$ (2D) E-TSP
  - TSP (E-TSP) is hard optimization problem $\rightarrow$ solution: approximation algorithms

TSP with triangle inequality

Fakts:

- MST is a natural **lower bound** for the length of the optimal route.
- In TSP with triangle inequality, it is possible to prove **upper bounds** in terms of the minimun spanning tree $\rightarrow$ 'Christofides Heuristics' [Christofides, 1976] ...

Minimum Spanning Tree (MST)

- Graph $G = (V, E, w)$ connected and attributed by weight w
  - $w : e \in E \rightarrow R^+$
- Goal : Find the spanning tree T with the smallest weight $\sum_{e \in T} w(e) \rightarrow \min$.
  - Easy optimization problem → solution: greedy algorithms
  - Kruskal's [Kruskal, 1956] and Prim's algorithms [Prim, 1957]
  - Borůvka's algorithm [Borůvka, 1926] ($\mathcal{O}(|E| \log |V|)$)

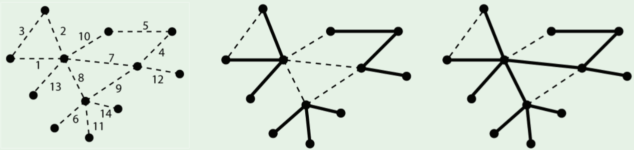
MST Algorithm
Borůvka's Version

MST Algorithm [Borůvka, 1926]

Input: graph $G = (V, E, w)$

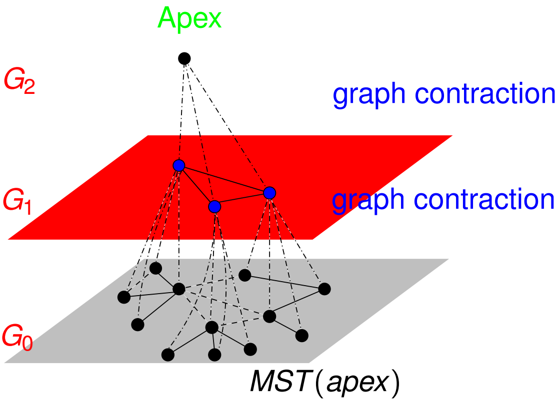
1: $MST :=$ empty edge list
2: $\forall v \in V$ make a list of trees L
3: **while** there is more than one tree in L **do**
4: each tree $T \in L$ finds the edge e with the minimum weight which connects T to $G \setminus T$ and add edge e to MST
5: using edge e merge pairs of trees in L
6: **end while**

Output: minimum spanning tree of G



Borůvka's Algorithm and Dual Graph Pyramid

- Graph contraction merges all trees $T \in L$ in step 3.
- Step 4: called Borůvka's step



Pyramid Solution

MST based approximate Algorithm for TSP

Input: graph $G = (V, E, w)$

while there is more than 3 cities **do**
    merge no more than k cites using Borůvka's step
end while
Find the trivial tour τ^*
repeat
    refine τ^*: $\tau \leftarrow \tau^*$
until the bottom of the pyramid

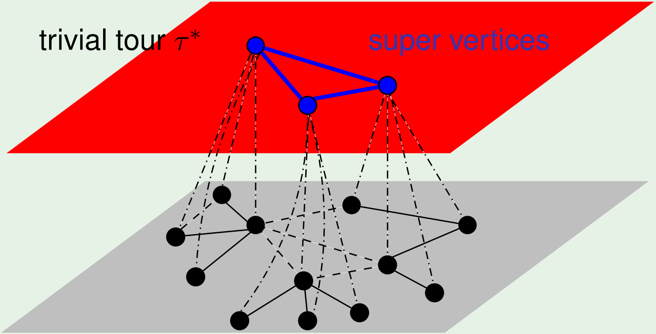
Output: (approximated) tour τ

$k \in \mathcal{N}$ and $k \geq 2$

+ invariant to shifts of the input
– Borůvka's step on fully connected graph $\rightarrow \mathcal{O}(|V|^2)$

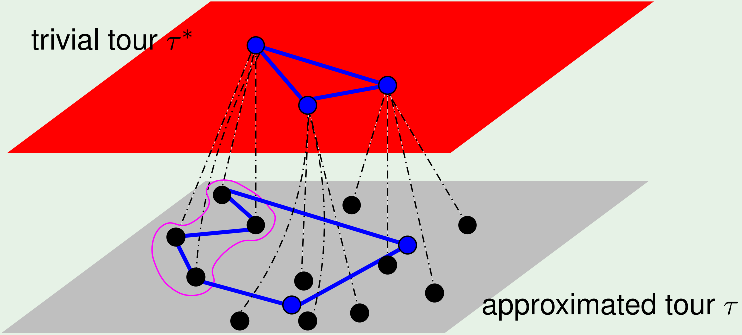
Top-down flow

Tour Refinement



Top-down flow

Tour Refinement



Solution Errors on Random Instances

MST Pyramid implementation issues

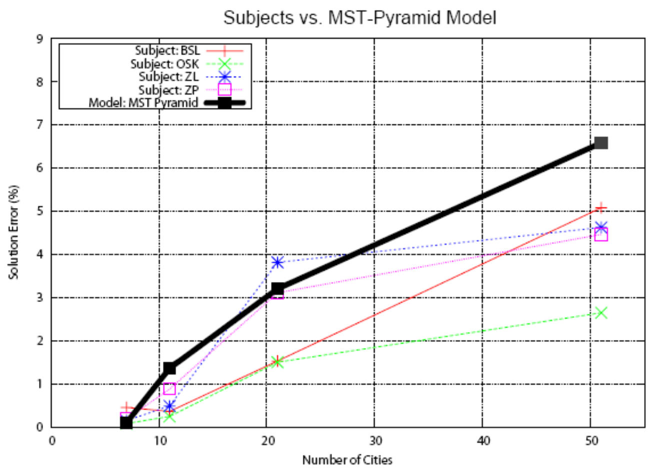
- $k = 7$
- super vertices on gravitational center of clusters

Tested on 6, 10, 20, 50 random instances

- Human subjects
- MST pyramid

Solution Errors on Random Instances

Subjects vs MST-based Pyramid



Comparing Solutions of Pyramid Algorithms

Compared

- Concorde algorithm [Applegate et al., 2001]
- Adaptive binary pyramid [Pizlo et al., 2006]
- MST-based pyramid

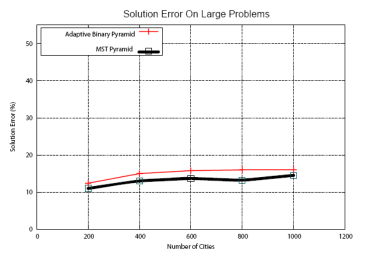
Input

- Random instances of 200, 400, 600, 800, 1000

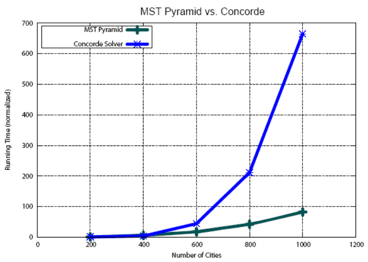
Code: http://bigbird.psych.purdue.edu/∼pizlo/ and http://www.tsp.gatech.edu/concorde/index.html

Comparison of the Algorithms

Pyramid Approximation



Solution Error



Running Time

Special TSP Instances

When subjects are tested on random instances they always produce close to optimal solutions

Hypothesis 1

Subjects minimize the total length of the tour

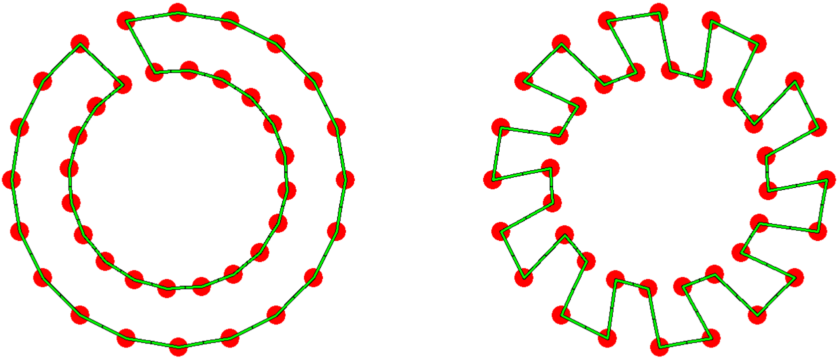
Hypothesis 2

Subjects optimize something else than the length
(this is how pyramid models work)

To tell between these two hypotheses nonrandom problems should be used?

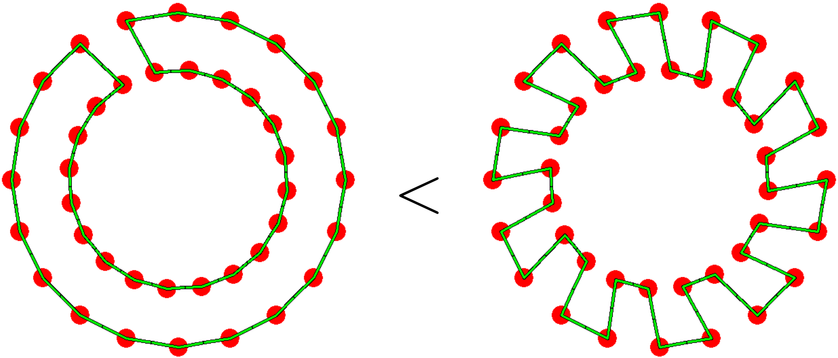
Two Rings ZigZag

40 city instance



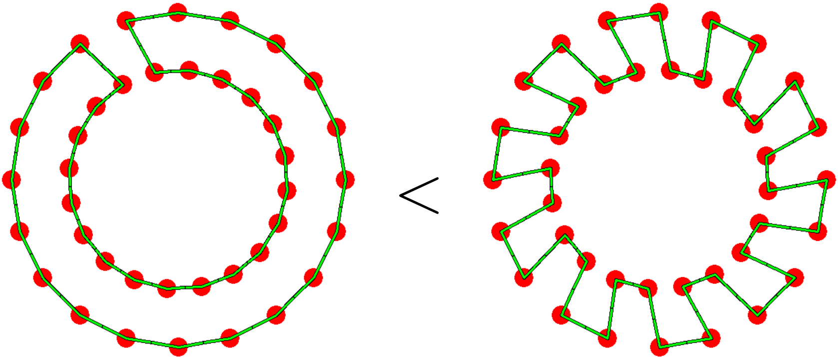
Two Rings < ZigZag

40 city instance



Two Rings < ZigZag

40 city instance



2596.42 $<$ 3013.6

16% difference

Optimal: 2594.07

Two Ring's length

- Two concentric circles with radii $R > r > 0$
- If density of cities along circle is high:
  1. the optimal tour follows one circle,
  2. switches then to the other circle
  3. which it follows in the opposite orientation
  4. and returns then back to the starting city.
- tour length (approx.): $L_R = 2\pi R + 2\pi r + 2(R - r)$

- Solution by nearest neighbor possible

ZigZag's length

- If density of cities along circle is low:
  1. the shorter tour follows the two circles
  2. jumping forth and back between the 2 circles in a zigzag fashion.
- tour length with n cities (approx.): $L_Z = \pi(R + r) + \frac{n}{2}(R - r)$

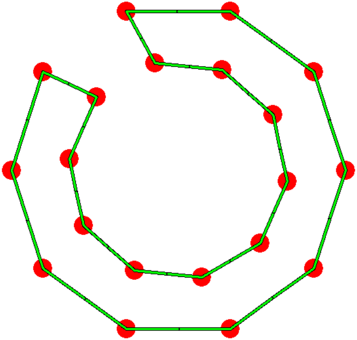
Two Rings = ZigZag

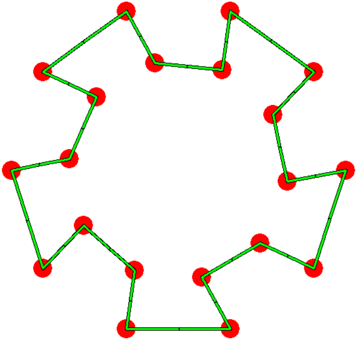
- If $L_R = L_Z$ both solutions have the same (optimal) tour length:
- let n_0 denote the number of cities for this case.
- Then $n_0 = 2 + 2\pi \frac{R+r}{R-r}$
-

$L_Z < L_R$		$L_R < L_Z$
$n < n_0$	$n = n_0$	$n > n_0$

Two Rings ZigZag

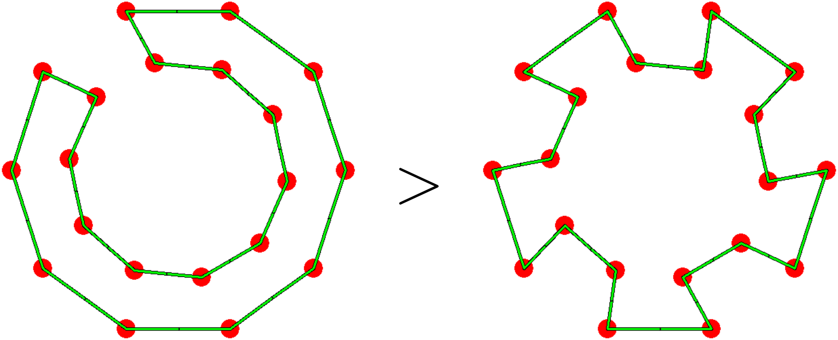
20 city instance





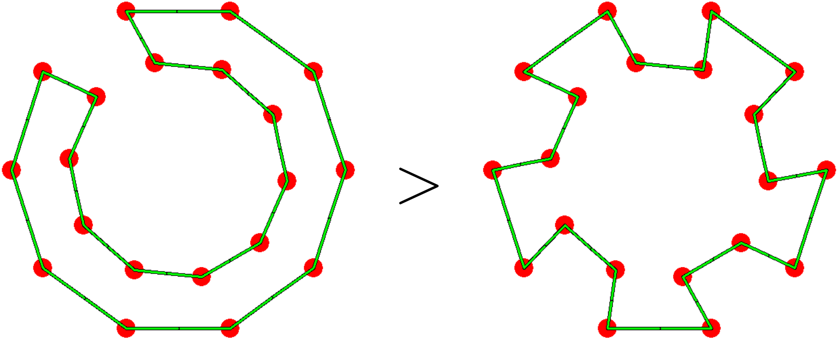
Two Rings > ZigZag

20 city instance



Two Rings > ZigZag

20 city instance

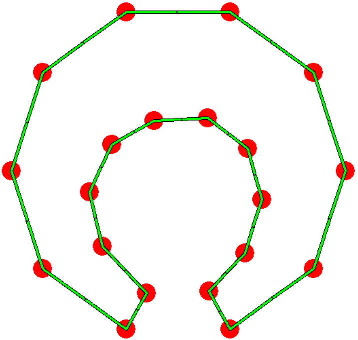


2440.28 > 2127.77

14% difference

Two non-concentric Rings $<$ ZigZag

20 city instance



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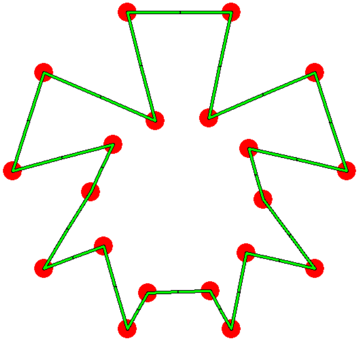

2209.15

2501.91

13% difference

Summary

- MST based TSP approximation algorithm
  - shows similar results as the other pyramid models on random instances
- Non-random instances suggest that humans do not minimize the total length of the tour

- Subjects and models will be tested on non-random instances
  - e.g. two circles with parameters: Radii R, r, number of cities n, circle offset

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