Technical Report

Dept. for Pattern Recognition and Image Processing
Institute for Automation
Technical University of Vienna
Treitlstr. 3/1832
A-1040 Vienna AUSTRIA
Phone: +43 (1) 58801-8161
Fax: +43 (1) 569697
E-mail: wil@prip.tuwien.ac.at

PRIP-TR-22 March 30, 1994

# Parallel Line Grouping in Irregular Curve Pyramids[1]

*Walter G. Kropatsch and Dieter Willersinn*

**Abstract**

Parallel lines are important features for object recognition by grouping. Regular $2 \times 2/2$ curve pyramids are hierarchical symbolic representations of curves that can be constructed and processed in logarithmic time. The rigidity of the regular structure causes an unstable, shift variant representation of parallel lines. In order to usefully apply the concept of the curve pyramid on grouping problems, the shift variance problem had to be overcome by extending the concept to irregular pyramids. These have a structure that adapts to the image data by deriving control information from curve relations. The algorithm that builds the irregular curve pyramid by deriving higher levels of abstraction from a set of relations goes far beyond merely solving the shift variance problem. It can reduce the computational complexity in comparable applications where all possible combinations of parts have to be checked in order to reassemble complex objects.

[1]This work was supported by the Austrian Science Foundation under grant P 8785

# 1 Introduction

Human observers can recognize objects almost independently of external influences, e.g. distance and orientation relative to the object. This is based on a process called perceptual organization. Lowe [14] describes perceptual organization as "referring to a basic capability of the human visual system to derive relevant groupings and structures from an image without prior knowledge of its content." Relevant groupings are made of image features that are *significantly related* to each others. These relations are significant if they are likely to be caused by an object in the scene rather than by accidental external conditions like for example the viewpoint of the observer.

Experiments have revealed the importance of greylevel discontinuities. Even if these are represented in simple line drawings, humans can easily perform object recognition, exploiting geometric relations between line segments ([3], loc. cit. [14]). Only geometric relations that are invariant against changes of the observer's viewpoint can have any significance for the presence of an object in the 3D scene. This limits drastically the number of relations to consider to only five perceptually significant classes (non-accidental properties, cf. [1]): straightness, connectivity, parallelism, curvilinearity, and lines meeting in a common point.

One method to accumulate evidence during perceptual grouping is referred to as *recursive application of structuring* by Lowe ([14], page 42f). This process can be implemented using the scheme of the $2 \times 2/2$ curve pyramid [5], the definition of which we summarize in Section 2. However, the regular curve pyramid has a severe drawback concerning one of the crucial relations used for perceptual organization: the representation of parallel lines is not stable with respect to the position in the image (Section 3). Section 4 contains basic definitions of pyramidal structures that can adapt to the image data, followed by Section 5 about how to adopt these notions for the construction of the irregular curve pyramid. Section 6 describes details of the solution of the shift variance problem, experimental results are reported in Section 7. Section 8 concludes this paper.

# 2 The regular curve pyramid

Straight lines and curves are connected sets of points. The connectivity of these sets is represented only implicitely in greylevel images. A more appropriate scheme for processing curves and lines is the curve pyramid, firstly introduced in [5]. We reproduce here the basic notions that will be used throughout the text.

**Definition 1** *A curve traverses the atomic cells of a two dimensional tesselation, intersecting with their sides. The* curve relation *is a binary relation between the intersected sides.*

**Definition 2** *A* curve image *is a two dimensional data structure built by the cells of the tesselation. It holds curve relations in its cells.*

**Definition 3** *A* curve pyramid *is a hierarchical sequence of curve images, derived from one single image (*the base*).*

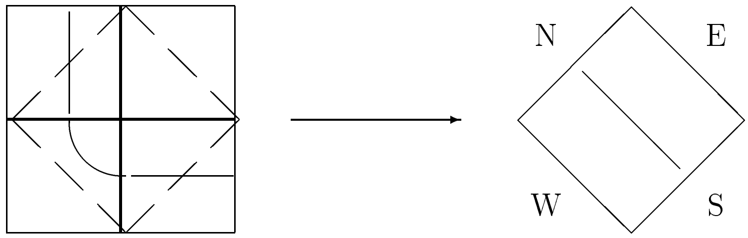


Figure 1: Reduction in the $2 \times 2/2$ curve pyramid

In image processing the classical arrangment of cells is a square grid. The $2 \times 2/2$ pyramid stacks square grids of successively lower resolution in a regular way: cells of overlapping $2 \times 2$ reduction windows are SPLIT by their diagonals, deriving curve relations in the resulting triangles. Groups of four triangles are then merged into one cell at the next level, building the transitive closure of the curve relations stored in the triangles (MERGE) [6]. Figure 1 shows in an example how the representation of a curve is simplified when a cell of the new pyramid level is constructed. The grid is rotated by 45 degrees, and the number

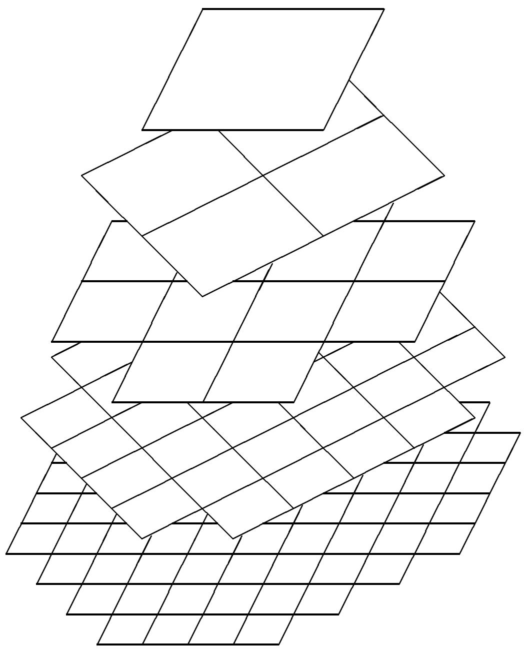

Figure 2: Structure of a $2 \times 2/2$ pyramid

of cells is divided by 2 from level to level. Figure 2 illustrates the structure of such a stack. A $2 \times 2/2$ pyramid recursively built with operations SPLIT and MERGE has been shown to possess the "length reduction property" [7]. It states that short curves remain at the lower (e.g. high resolution) levels of this "curve pyramid" whereas long curves survive up to higher (e.g. low resolution) levels. The connectivity is preserved in the bottom-up building process.

Two applications have demonstrated the efficiency of the concept: structural filtering of short curves produced by noise [8, 10, 11] and preserving the contrast of boundaries in the concept of dual pyramids [18, 9].

# 3 The problem of representing parallel lines

Unfortunately the $2 \times 2/2$ curve pyramid has also some drawbacks related to its rigid structure. These drawbacks are similar in nature to those described by Bister et al. [2].

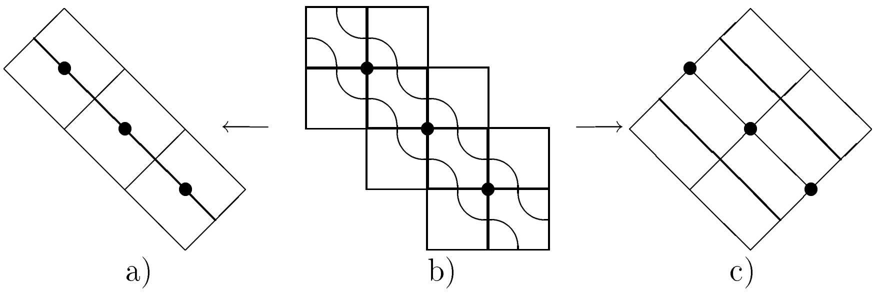

Figure 3: Curves may or may not be merged

One of the drawbacks of the $2 \times 2/2$ curve pyramid involves the representation of parallel lines. The notion of parallelity is defined only with respect to continuous coordinates.

**Definition 4** *Two lines are* parallel *if they are straight, and if they are at the same distance to each others in any point.*

After the sampling process, lines are encoded in terms of the sides they intersect with, which is only an approximation of their geometric location. However, we don't need the notion of parallelity to extract long lines in the curve pyramid. All we have to assure is that lines continue to be represented until they are covered entirely by one single cell of an image within the curve pyramid. This must be true whatever the position of the representation in the curve image is. Two parallel lines at 45 degree and at a distance of one pixel (Figure 3 b) are represented by two series of opposite left-right turns. If the vertices (•) between the two curves survive (Figure 3 c), both lines will be represented at the next two lower resolutions yielding the same situation as before. However if the vertices between the curves do not survive the two lines will be represented by one (double) curve (Figure 3 a). Pyramidal structures that show instabilities of that kind are unusable for a modeling of perceptual grouping, given the importance of pairs of parallel lines for cognitive processes.

# 4 Elements of the solution: irregular, adaptive structures

In Section 3, we have stated the problem of unstable object representation in the regular curve pyramid. Pyramidal structures that are flexible enough to adapt their structure during construction have been introduced by P. Meer [15]. Levels within these structures are no longer regular grids, but general graph structures. We summarize the basic notions of irregular and adaptive structures in definitions 5 through 8.

**Definition 5** Stochastic decimation *of a graph is a process that is executed in the following steps:*

1. *Random numbers are assigned to the vertices.*
2. *Vertices with a local maximum of this variable (*surviving vertices*) are selected.*
3. *All* non-surviving vertices *are assigned to one survivor out of their neighborhood.*
4. *The* receptive field *of a survivor is formed by all non-survivors that are assigned to this survivor. It also includes the survivor itself.*
5. *Neighbor-relations of the reduced graph are determined by the receptive fields and neighbor-relations of the original graph.*

**Definition 6** *A* stochastic pyramid *is a stack of general graph structures with a decreasing number of vertices. It is constructed from level to level by recursive execution of stochastic decimation.*

**Definition 7** *In an* adaptive pyramid *the selection of survivors is related to the image data [4, 17, 16].*

**Definition 8** Irregular pyramids *is a generic term that refers to the fact that in stochastic as well as in adaptive pyramids neighbor-relations are not regular.*

The scheme of the adaptive pyramid overcomes many of the drawbacks of rigid pyramids because the structure of the pyramid reflects the structure of the data.

# 5 Adopting the elements to the curve pyramid

We start adopting irregular structures by defining a graph representation of a curve image.

**Definition 9** *A* graph representation of a curve image *consists of* faces *that are connected via* sides. *Faces are surrounded by sides, and curve relations are stored in faces, encoding binary relations between sides surrounding the faces.*

One might think of recursively decimating such a graph representation of curve images to build an irregular curve pyramid. Unfortunately, this causes problems which are due to a property of the stochastic decimation process firstly proofed in [12]:

**Lemma 1** *The degree of a graph does not remain bounded during recursive stochastic decimation.*

**Corollary 1** *If a graph representation of a curve image is decimated, then the number of sides of the faces will not be bounded. Therefore also the number of possible curve relations in a face and hence the space to unambiguously store these relations in the faces will not be bounded.*

Besides the space problem mentioned in corollary 1 the unboundedness of the face degree also implies an unlimited number of neighbors of a face, with the consequence of unlimited time complexity of local operations. Faced with these problems, dual graphs were taken into account: we decimated a neighborhood graph, the vertices of which span the faces of a curve image. From the decimated vertices, new faces were constructed. We proofed that the degree of the thus constructed faces is less or equal to the degree of the original faces[12].

Given this result, we can now enumerate the elements of the irregular curve pyramid.

**Definition 10** *We build the irregular curve pyramid the levels of which are a dual graph representation consisting of the following elements:*

1. *A* neighborhood graph $G(V, E)$ *represents* vertices $v \in V$ *that span faces. Vertices of this graph are connected by* edges $e \in E$.
2. *Faces are represented in the dual of* $G(V, E)$, $\overline{G}(F,\overline{E})$, *with faces* $f \in F$ *and sides* $\overline{e} \in \overline{E}$.

We formally distinguish between the edges $e \in E$ and sides $\overline{e} \in \overline{E}$. However, there is a one-to-one match between sides and edges, and their graphical representation is the same line segment.

**Definition 11** *The* levels of the pyramid *are counted bottom up, with the base being level* 0 *of the pyramid. For graphs representing a level* $n$ *of the pyramid we use a notation with an index* $n$*:* $G_n(V_n, E_n)$, $\overline{G_n}(F_n,\overline{E_n})$.

In a level $n$ of the pyramid, we decimate the vertices of $G_n(V_n, E_n)$ to obtain the vertices of level $n + 1$, and then construct edges, sides and faces from $V_n$ as described in [13]. The degree of the faces in a thus constructed graph $\overline{G_{n+1}}$ will be less or equal the degree of faces in $\overline{G_n}$.

# 6 Solving the problem

We come to the central theorem of this paper. We start by defining the distance between two curves as well as inner and outer vertices with respect to this pair.

**Definition 12** *Consider two curves $A$ and $B$ and the sets of sides $\overline{E_A}$, $\overline{E_B} \subset \overline{E}$ they intersect with. The* topological distance *between curves $A$ and $B$ is equal to the number of vertices on the shortest path that connects any side of $\overline{e_A} \in \overline{E_A}$ to a side $\overline{e_B} \in \overline{E_B}$. Vertices on these paths are called* inner vertices *with respect to the pair of curves, all other vertices are called* outer vertices.

Figures 3 and 4 show examples of curves at distance one and two, respectively. The

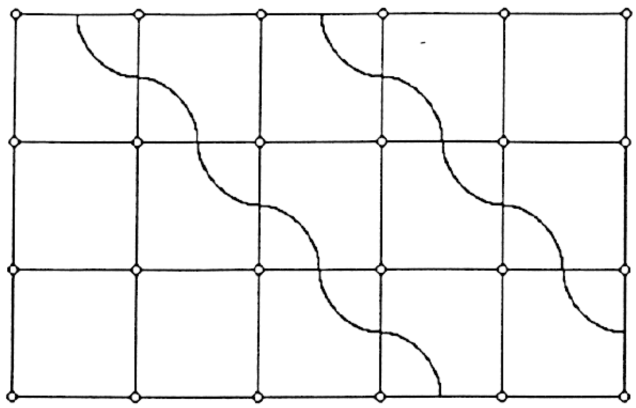

Figure 4: Curves at distance two

conditions for the survival of a pair of curves can now be stated.

**Lemma 2** *Curves are merged at the next level of the pyramid, whenever there will be no code difference between the curves, e.g. if only sides are created that intersect with both curves. They are preserved if survivors from outside the pair are connected via a survivor between the curves.*

**Proof :** If no survivor is lying between two curves, then survivors at the next level of the pyramid will be connected by edges intersecting with both curves: only one curve relation is generated. Whenever a survivor is lying between the two curves, sides will be created that are intersected by only one curve. Hence the two curves will be encoded differently at the next level. ■

**Lemma 3** *Before a decimation we can mark the importance of vertices by distributing scores in faces of the dual graph $\overline{G_n}$. The scores are distributed over the vertices that span the sides of the faces. Vertices with the highest score, e.g. the most important ones, will then survive the decimation process.*

**Proof :** Vertices of a neighborhood graph survive in a stochastic decimation process if the value of their random variable represents a local maximum. The random numbers added to the scores before the decimation are within $[0, 1)$. If they do not exceed the quantization step of the score distribution, the selection of survivors implied by scores remains unchanged.However, they are used to break the remaining ties where vertices bear equal scores. ■

We have proofed in Lemma 2 that the survival of a pair of curves depends on survivors between the curves. We also have shown that we can force the most important vertices to survive by means of scores that we distribute in faces (Lemma 3). Consequently, scores need to be distributed in all faces that hold curve relations, in a way that vertices between pairs of curves obtain the highest scores out of their neighborhood. Since this will be based on local processes, we need more precise ideas about possible constellations.

**Lemma 4** *Only curves at a distance of one or two may be merged during a stochastic decimation process as described in Section 4.*

**Proof :** If the distance between two curves is three (or greater), then survivors from either side of the pair can't be connected at the next level of the pyramid, given that the decimation is carried out as described in Section 4. Thus the problem of two curves being merged during decimation can occur only if the curves are at distance one or two. ■

In an irregular curve pyramid that starts with square faces in the base level, only faces with three and four sides can occur. To derive scores from curve relations we only need to consider three basic configurations and combinations of them. These basic configurations are shown in Figure 5, along with the scores that we distribute to control the decimation process. The idea behind scoring is to equally score vertices on either side of a curve. The distribution indicated in Figure 5 b) and c) locally contradicts this for the sake of the *length reduction* of the irregular curve pyramid [13]. However, the equal score distribution on both sides of a curve is preserved.

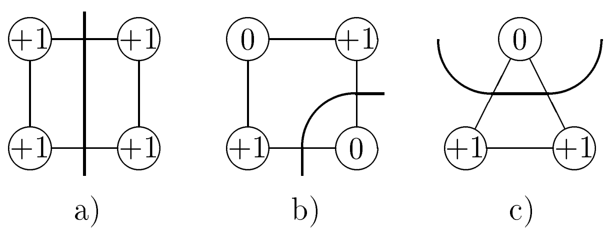


Figure 5: Scores are distributed in faces with a curve relation

## 6.1 Curves at distance one

For curves at distance one we state Lemma 5. For an illustration see Figure 6.

**Lemma 5** *Vertices between curves at distance one accumulate scores distributed due to both curves. If no other curve lies within a distance of two from either of the two curves, then the inner vertices obtain higher scores than vertices outside the pair.*

**Proof :** Vertices between curves at distance one are common vertices for the sides being intersected by both curves. Hence they are also common vertices for faces that hold representations of both curves. Thus they are the only vertices being scored due to the representations of both curves $A$ and $B$. ■

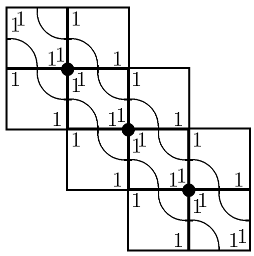

Figure 6: Score distribution for the example of parallel lines at 45 degrees

## 6.2 Curves at distance two

In the case of curves at distance two, simply distributing scores in faces with curve relations does not assure the survival of the pair as such.

**Lemma 6** *Vertices between a pair of curves at distance two cannot accumulate scores distributed due to both curves. Therefore they can't bear higher scores than vertices outside the pair.*

**Proof :** By Definition 12, the faces traversed by two curves at distance two do not have any common vertex. Therefore, no vertex in between the two curves can obtain scores that are due to the representation of both curves. ■

Figure 7 illustrates how vertices inside and outside a pair of curves are equally scored. However, scores on the vertices around faces can be used to detect the presence of a pair of curves at distance two.

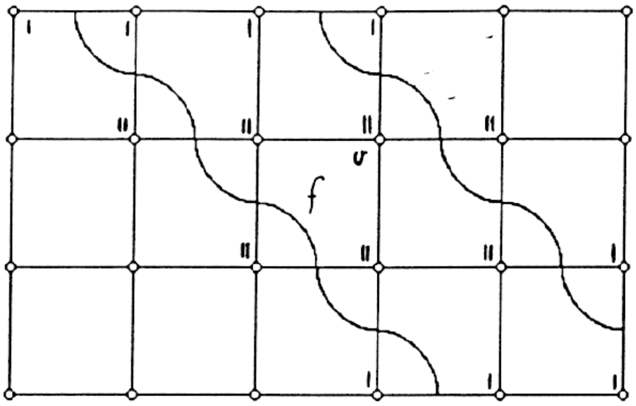


Figure 7: Example for score distribution in the case of curves at distance two

**Lemma 7** *Consider a vertex where two sides of a face meet. If neither of these sides is curve related, and the vertex bears a score, then it indicates the presence of a curve that is not represented in the face we consider.*

**Proof :** Any vertex of a face that holds "foreign" scores not distributed in the face itself indicates the presence of another face with curve relations. However, if these vertices are joined by sides that are curve related, then their additional scores come from a continuation of the curve already represented in the face. ■

For an illustration, see Figure 7. In face $f$, scores on vertex $v$ indicate the presence of another curve. Vertex $v$ is identified as being in between two curves at distance two.

**Lemma 8** *Faces in between two curves at distance two may be free of curve relations. However, all vertices spanning these faces bear a score.*[1]

**Proof :** All vertices between two curves are scored. If the curves are at distance two, then these vertices may span faces without curve relation. ■

For an illustration of two curves with "empty" faces between them see Figure 8.

In Lemmas 7 and 8 we have shown how to identify vertices between curves at distance two. These indications give us the means to prevent curves at distance two from being merged: the scores on thus identified vertices are incremented during an additional process.

## 6.3 The solution

Having gathered all elements of the solution, we can now state the central theorem of this paper.

[1]Note that scores on all corners of a face in the absence of curve relations are a necessary, but not a sufficient condition for adjacent curves. However, this feature can be used to detect vertices between a pair of curves at distance two.

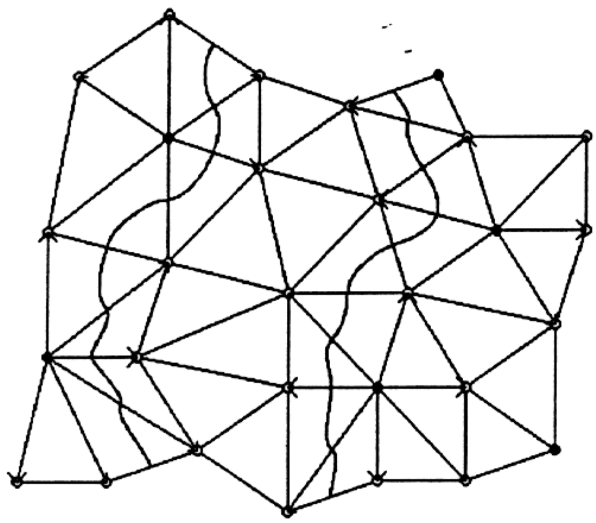

Figure 8: Curves at distance two, with faces bearing no curve relations in between.

**Theorem 1** *In an irregular curve pyramid parallel lines are preserved independently of their position.*

**Proof :** Parallel lines are represented in a discrete space as a pair of curves. In Lemma 2 we have set the conditions for a pair of curves to survive: vertices between them must survive. In Lemma 3 we have proofed that we can influence the survival of vertices by means of scores. Furthermore, we have discussed the two possible cases of curves at distance one and two in sections 6.1 and 6.2. The scores distributed depend entirely on curve relations in faces, and not on a whatever absolute position of the object. Therefore we can assure the survival of a pair of curves as such independently of its position in the tesselation. ■

# 7 Experimental results

The figures in this section correspond pairwise to each others and represent two stages of the decimation process. The first figure of each pair shows a subgraph of a level $n$ of the irregular curve pyramid, with the selection of survivors (•) and non-survivors (o) already completed. Arcs pointing from survivors to non-survivors indicate the receptive field assignment. The second figure of a pair shows the corresponding subgraph at level $n+1$. Figure 9 shows another example of a pair of curves at distance one (cf. Figure 3). Scoring on either side of the curves accumulates more scores on inner vertices than on vertices outside the pair: vertices between the curves survive, both curves are preserved (Figure 10).

Figures 11 and 12 demonstrate what can happen if curves are at distance two. Scoring in faces that are traversed by the curves cannot accumulate more scores on inner than

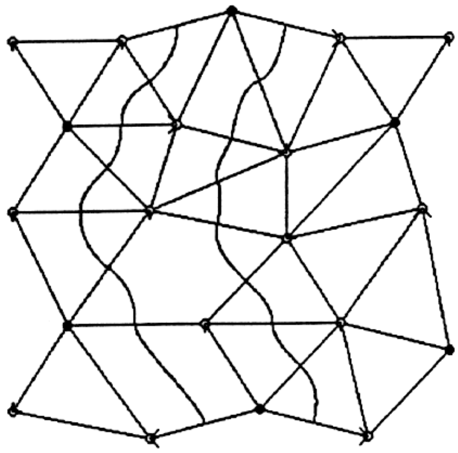

Figure 9: Vertices between curves at distance one survive the decimation process

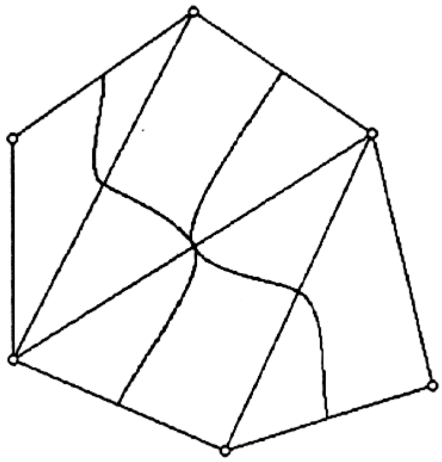

Figure 10: Two curves at distance one are preserved at the next level

on outer vertices. Without the additional process described in Section 4 all inner vertices can become non-survivors (Figure 11), and the curves are merged into one single curve at the next level (Figure 12). Figures 13 and 14 show the efficiency of the additional scoring process defined in Section 6.2: vertices with "foreign" scores are detected and obtain additional scores as described in Section 4. The two curves are preserved from being merged.

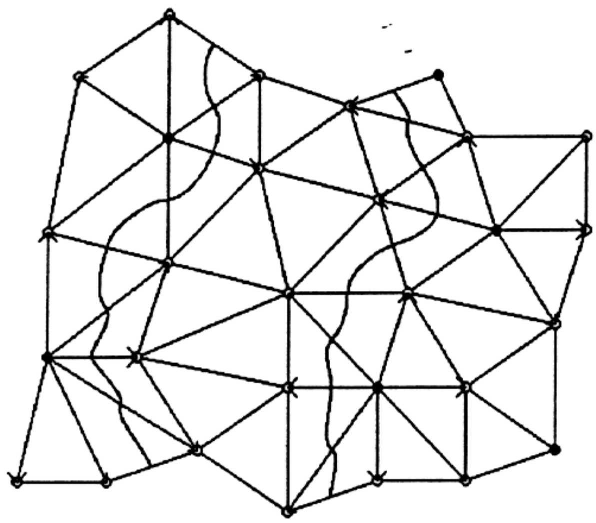

Figure 11: Curves at distance two, with none of the inner vertices surviving

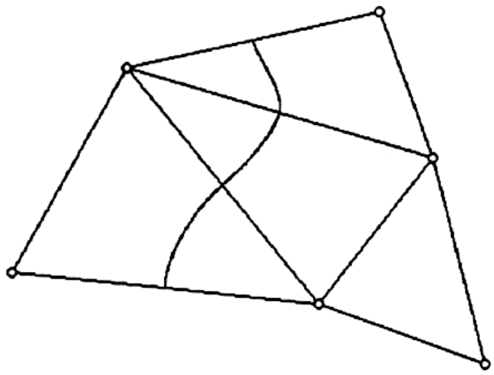

Figure 12: Two curves at distance two, merged at the next level

# 8 Conclusion

The position variance problem mentioned in Section 3 has been solved: by means of the scores we can avoid the merge of two parallel lines. However there are exceptions in the case where more than two parallel lines are present in the scene: we cannot avoid the merge of lines. This is a constellation where the "ratio of background density to proximity" [14] also sets a limit to recognition by the human visual system. To be sure to identify objects correctly in the curve pyramid we require that objects be distinct from their surrounding in all scales we consider. Although motivated by the goal of finding long connected curves in digital images the scope of the presented approach goes beyond the original goal. Putting together curve segments that meet in a common point has many analogies in image analysis

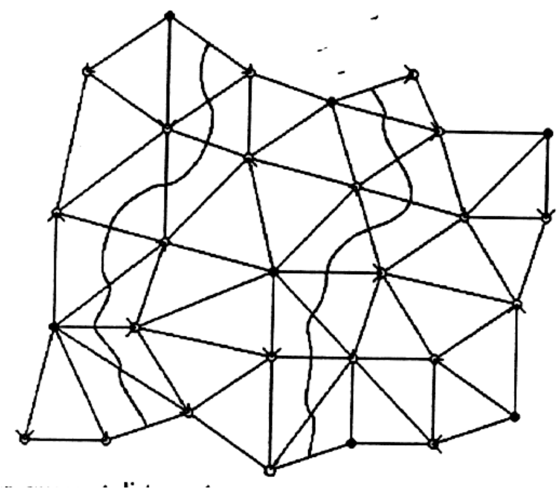

Figure 13: Survivors between curves at distance two

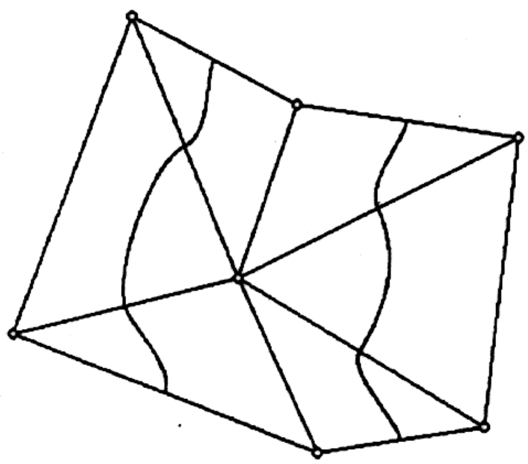

Figure 14: Two curves at distance two are preserved

as in other fields of computer science. For example, a complex object is often composed of several parts which themselves are composed of smaller parts. These object parts have different physical properties and, hence, may be recognized as individual image parts. In order to reassemble an object from its parts, pairs (or triples ...) of parts must satisfy certain constraints, e.g. they meet at a given angle. Checking all possible combinations of parts is a problem of high computational complexity ("combinatorial explosion"), as is the problem of building the transitive closure of a set of relations. Our approach considerably reduces this complexity by

1. embedding the set into a discrete partition of a (geometrical) space,

2. building a hierarchy of partitions using only local processes to aggregate small parts to larger parts,
3. adapting the structure of this hierarchy to the image data to overcome certain problems arising in rigid (regular) structures.

# References


[1] Irving Biederman. Matching image edges to object memory. In *Proceedings of the First International Conference on Computer Vision*, pages 384–392, London, England, 1987.

[2] M. Bister, J. Cornelis, and Azriel Rosenfeld. A critical view of pyramid segmentation algorithms. *Pattern Recognition Letters*, Vol. 11(No. 9):pp. 605–617, September 1990.

[3] J.E. Hochberg and V. Brooks. Pictoral recognition as an unlearned ability: A study of of one child's performance. *American Journal of Psychology*, 75:624–628, 1962.

[4] Jean Michel Jolion and Annick Montanvert. The adaptive pyramid, a framework for 2D image analysis. *Computer Vision, Graphics, and Image Processing: Image Understanding*, 55(3):339–348, May 1992.

[5] Walter G. Kropatsch. A pyramid that grows by powers of 2. *Pattern Recognition Letters*, Vol. 3:pp.315–322, 1985.

[6] Walter G. Kropatsch. Kurvenrepräsentation in Pyramiden. In Walter G. Kropatsch and P. Mandl, editors, *Mustererkennung'86*, OCG-Schriftenreihe, Österr. Arbeitsgruppe für Mustererkennung, pages 16–51. Oldenbourg, 1986. Band 36.

[7] Walter G. Kropatsch. Curve representations in multiple resolutions. *Pattern Recognition Letters*, Vol. 6(No. 3):pp.179–184, August 1987.

[8] Walter G. Kropatsch. Elimination von "kleinen" Kurvenstücken in der $2 \times 2/2$ Kurvenpyramide. In E. Paulus, editor, *Mustererkennung 1987*, Informatik Fachberichte 149, pages 156–160. Springer Verlag, 1987.

[9] Walter G. Kropatsch. Preserving contours in dual pyramids. In *Proc. 9th International Conference on Pattern Recognition*, pages 563–565, Rome, Italy, November 1988. IEEE Comp.Soc.

[10] Walter G. Kropatsch. Digitales Sehen mit Bildpyramiden. *Elektronik-Journal: Maschinelles Sehen - Industrielle Bildverarbeitung*, pages pp.93–102, 1990.

[11] Walter G. Kropatsch. Hierarchical methods for robot vision. In T. Jordanides and B. Torby, editors, *Expert Systems and Robotics*, volume 71, pages 63–109, NATO-ASI Corfu, Greece, 1991. Springer. Series F: Computer and Systems Sciences.

[12] Walter G. Kropatsch and Annick Montanvert. Irregular versus regular pyramid structures. In U. Eckhardt, A. Hübler, W. Nagel, and G. Werner, editors, *Geometrical Problems of Image Processing*, pages 11–22, Georgenthal, Germany, March 1991. Akademie Verlag, Berlin.

[13] Walter G. Kropatsch and Dieter Willersinn. Representing curves in irregular pyramids. In W. G. Kropatsch and H. Bischof, editors, *Pattern Recognition 1992*, pages 333–348, Vienna, Austria, May 1992. Oldenbourg.

[14] David G. Lowe. *Perceptual Organisation and Visual Recognition*. Kluwer, 1985.

[15] Peter Meer. Stochastic image pyramids. *Computer Vision, Graphics, and Image Processing*, Vol. 45(No. 3):pp.269–294, March 1989.

[16] Annick Montanvert and Pascal Bertolino. Irregular pyramids for parallel image segmentation. In W. G. Kropatsch and H. Bischof, editors, *Pattern Recognition 1992*, pages 13–34, Vienna, Austria, May 1992. Oldenbourg.

[17] Annick Montanvert, Peter Meer, and Azriel Rosenfeld. Irregular tesselation based image analysis. In *Proc. 10th International Conference on Pattern Recognition*, pages 474–479, Atlantic City, New Jersey, USA, June 1990. IEEE Comp.Soc. Vol. I.

[18] G. Paar and Walter G. Kropatsch. Hierarchical cooperation between numerical and symbolic image representations. In R. Mohr, Theo Pavlidis, and A. Sanfeliu, editors, *Structural Pattern Analysis*, pages 113–130. World Scientific Publ. Co., 1988.