A revision of pyramid segmentation*

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Abstract

Dual graph contraction reduces the number of vertices and of edges of a pair of dual image graphs while, at the same time, the topological relations among the 'surviving' components are preserved. Repeated application produces a stack of successively smaller graphs: a pair of dual irregular pyramids. The process is controlled by selected decimation parameters which consist of a subset of surviving vertices and associated contraction kernels. Equivalent contraction kernels (ECKs) combine two or more contraction kernels into one single contraction kernel which generates the same result in one single dual contraction. This is the basis to the proof that any segmentation can be represented in one single level of such a pyramid. Experimental results demonstrate the applicability on synthetic and real images respectively.

1 Introduction

Pyramids, considered as a stack of interrelated images with decreasing resolution, are useful when dealing with image processing tasks and feature extraction. Among the inherent properties of pyramids [7] are: reduction of noise and computational cost, resolution independent processing, processing with local and global features within the same frame, irregular pyramids adapt their structure to the data.

In the paper of Bister *et al.* [1], properties of pyramid segmentation algorithms were investigated. The main conclusion of the paper is that *" classical pyramids have to be rejected as general-purpose image segmentation algorithm, due to several problems such as shift-variance"*. In the same paper Bister *et al.* reject the use of irregular pyramids because *"we do not know how many levels we will have, thus e.g, eliminating the use of pyramid architectures for implementation of this approach.* Many attempts were undertaken to overcome the problem of classical pyramid segmentation algorithms by using irregular structures [10, 9, 11]. We propose to address the problem of pyramid segmentation algorithm using a unified scheme that embeds the already cited approaches and that overcomes their problem. This will allow us to have a shift-invariant pyramidal segmentation algorithm with a bounded number of levels for the pyramid. Moreover we obtain an algorithm that is invariant under all transformations preserving connectivity. The presented approach addresses a representation of pure structure, a hierarchy of plane graphs, with a clear interface, the decimation parameters, to control generation and modification of the structure. Dual graph contraction is the basic process [5] that builds an irregular "graph" pyramid by successively contracting a dual image graph of one level into the smaller dual image graph of the next level. Dual image graphs are typically defined by the neighborhood relations of image pixels or by the adjacency relations of the region adjacency graph. The above concept has been used for finding the structure of connected components [8]. It also embeds other approaches ([10], [3], [9]).

The paper is organized as follows. We first summarize and illustrate the procedure of dual graph contraction in Section 2. The observation that the parameters that control the process form forests is then generalized by the concept of contraction kernels. Originally of depth one, deeper forests are now permitted and allow bigger contractions. They are necessary if repeated dual contractions are to be replaced by a single dual contraction using equivalent contraction kernels (ECK) (section 2.2). ECKs are able to compute any level of an irregular pyramid directly from the base. Decimation parameters can be designed now at the base without the need to first generate the lower pyramid levels. The main properties of ECK are used in section 3 to derive the main result of this paper. The results in section 4 include an example from Bister *et al.* [1] that demons-

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trates shift-variance with regular pyramid linking and is stable in the new scheme.

2 Dual Graph Contraction

The base of the pyramid consists of the pair of dual image graphs $(G_0, \overline{G_0})$. We repeat the definition of the parameters determining the structure of an irregular pyramid given in [5][Def.5]:

Definition 1
In a pair of dual image graphs $(G_i(V_i, E_i), \overline{G_i}(\overline{V_i}, \overline{E_i}))$, *following* **decimation parameters** $(S_i, N_{i,i+1})$ *determine the contracted graphs* $(G_{i+1}, \overline{G_{i+1}})$*: a subset of* **surviving vertices** $S_i = V_{i+1} \subset V_i$*, and a subset of* **primary non-surviving edges**[1] $N_{i,i+1} \subset E_i$*. Every non-surviving vertex,* $v \in V_i \setminus S_i$*, must be connected to one surviving vertex in a unique way:*

$$\forall v \in V_i \setminus S_i \quad \exists s \in S_i : (v, s) \in N_{i,j}. \tag{1}$$

The relation between the two pairs of dual graphs, $(G_i, \overline{G_i})$ *and* $(G_{i+1}, \overline{G_{i+1}})$*, as established by dual graph contraction with decimation parameters* $(S_i, N_{i,i+1})$ *is expressed by function* $C[.,.]$*:*

$$(G_{i+1}, \overline{G_{i+1}}) = C[(G_i, \overline{G_i}), (S_i, N_{i,i+1})] \tag{2}$$

For more details see [13].

2.1 Decimation with Contraction kernels

Definition 2 *A decimation of a graph* $G(V, E)$ *is specified by a selection of* **surviving** *vertices* $S \subset V$ *and a selection of* **primary non-surviving edges** $N \subset E$ *such that following two conditions are fulfilled:*

1. *Graph* (V, N) *is a spanning forest of graph* $G(V, E)$*.*
2. *The surviving vertices* $S \subset V$ *are the roots of the forest* (V, N)*.*

The trees $T(v)$ *of the forest* (V, N) *with root* $v \in V$ *are called* **contraction kernels**.

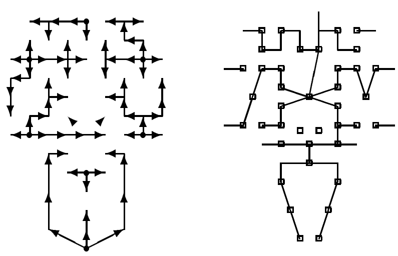
Instead of joining non-surviving vertices by an edge to their corresponding surviving parent vertex, the new concept establishes this connection via paths of non-surviving edges (e.g. branches of the trees). The concept of connecting path as introduced in [5][Def.6] is adapted accordingly:

[1] Secondary non-surviving edges are removed during dual face contraction.

Definition 3 *Let G(V,E) be a graph with decimation parameters* (S, N)*. A path in* $G(V, E)$ *is called a* **connecting path** *between two surviving vertices* $v, w \in S$*, denoted* $CP(v, w)$*, if it consists of three subsets of edges E:*

1. *The first part is a possibly empty branch of contraction kernel* $T(v)$*.*
2. *The middle part is an edge* $e \in E \setminus N$ *that bridges the gap between the two contraction kernels* $T(v)$ *and* $T(w)$*. We call e the* **bridge** *of the connecting path* $CP(v, w)$*.*
3. *The third part is a possibly empty branch of contraction kernel* $T(w)$*.*

Connecting paths $CP(v, w)$ in $G(V, E)$ are strongly related to the edges in the contracted graph $G'(V', E')$: Two different surviving vertices that are connected by a connecting path in G are connected by an edge in E'. For every edge $e' = (v, w) \in E'$ there exists a connecting path $CP(v, w)$ in G. Dual edge contraction can be implemented by (1) simply renaming all the non-surviving vertices to their surviving parent vertex, (2) deleting all non-surviving edges N and (3) their duals $\overline{N}$.



(a) $(V_2, N_{0,2})$ (b) $(\overline{V_0}, \overline{E_0} \setminus \overline{N_{0,2}})$

Figure 1. Example of equivalent contraction kernel.

Fig. 1a shows different decimation parameters: Survivors $S = V_2$ are selected and the contraction kernels $N_{0,2}$ cover G_0. Like in a maze the edge-contracted face graph (Fig. 1b), $\overline{G_0^*}(\overline{V_0}, \overline{E_0} \setminus \overline{N_{0,2}})$, fills in the holes left between the contraction kernels. Dual face contraction deletes all degree-one faces and shortens redundant connections established by the degree-two faces, resulting in $\overline{G_2}$.

2.2 Equivalent contraction kernels

Burt [2] introduced the 'equivalent weighting function': *"Iterative pyramid generation is equivalent to convolving the image g_0 with a set of 'equivalent weighting functions' h_l:"* $g_l = h_l * g_0 = h * g_{l-1}, l > 1$. It allowed him to study the effects of iterated reduction (e.g. the low-pass character of Gaussian pyramids) using the single parameter h_l without giving up the efficient iterative computation.

$$C[C[G_{k-2},(S_{k-2},N_{k-2,k-1})],(S_{k-1},N_{k-1,k})] \quad = \quad G_k$$
$$G_k = C[G_{k-2},(S_{k-1},N_{k-2,k})] \qquad (3)$$

Equivalent contraction kernels are constructed in the following way:

Assume that the dual irregular pyramid $((G_0,\overline{G_0}),(G_1,\overline{G_1}),\ldots,(G_k,\overline{G_k}))$, $k > 1$, is the result of k dual graph contractions. The structure of G_k is fully determined by the structure of G_{k-1} and the decimation parameters $(S_{k-1}, N_{k-1,k})$. Furthermore, the structure of G_{k-1} is determined by G_{k-2} and the decimation parameters $(S_{k-2}, N_{k-2,k-1})$. $S_{k-1} := V_k$ are the vertices surviving from G_{k-2} to G_k. The searched contraction kernels must be formed by edges $N_{k-2,k} \subset E_{k-2}$. This is true for $N_{k-2,k-1} \subset E_{k-2}$ but not for the contracted edges, e.g. $N_{k-1,k} \not\subset E_{k-2}$. An edge $e_{k-1} = (v_{k-1}, w_{k-1}) \in N_{k-1,k}$ corresponds to a connecting path[2] $CP(v_{k-1}, w_{k-1})$ in G_{k-2}. By definition 3, $CP(v_{k-1}, w_{k-1})$ consists of one branch of $T_{k-2}(v_{k-1})$, one branch of $T_{k-2}(w_{k-1})$, and one surviving edge $e_{k-2} \in E_{k-2}$ connecting the two contraction kernels $T_{k-2}(v_{k-1}), T_{k-2}(w_{k-1})$.

Definition 4 *Function* **bridge**: $E_{k-1} \mapsto E_{k-2}$ *assigns to each edge* $e_{k-1} = (v_{k-1}, w_{k-1}) \in E_{k-1}$ *one of the bridges* $e_{k-2} \in E_{k-2}$ *of the connecting paths* $CP(v_{k-1}, w_{k-1})$*:*

$$\mathit{bridge}(e_{k-1}) := e_{k-2}. \qquad (4)$$

Two disjoint tree structures connected by a single edge become a new tree structure. The result of connecting all contraction kernels T_{k-2} by bridges fulfills the requirements of a contraction kernel:

$$N_{k-2,k} := N_{k-2,k-1} \quad \cup \bigcup_{e_{k-1} \in N_{k-1,k}} \mathrm{bridge}(e_{k-1}) \quad (5)$$

The above process can be repeated on the remaining contraction kernels until the base level 0 contracts in one step into the apex $V_n = \{v_n\}$. The edges of the corresponding spanning tree are contained in $N_{0,n}$. Fig. 2 shows spanning tree $N_{0,4}$ overlaid with the base graph G_0. The apex, $v_4 \in V_4$, is marked by a filled circle and the edges of the spanning tree $N_{0,4}$ are differentiated from edges E_0 by triple lines.

[2] If there are more than one connecting paths, one must be selected.

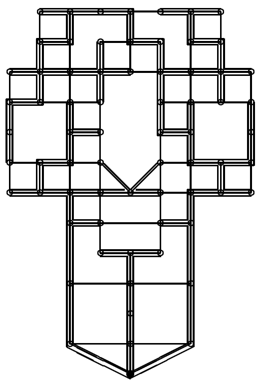


Figure 2. Example of ECK of apex: $G_0 \cup N_{0,4}$.

3 Contracting a segmentation set

One step to recover the structure of the scene from the projected structure in the image is to find the adjacency relations between the regions of a segmentation [8]. Pyramids are computationally very efficient and can be used for segmentation. (details in [4]).

Pyramid (re-)linking allows a fine tuning of region shapes by removing certain cells of the receptive field because they link to another ancester. But the resulting regions always fit inside the original region. Besides the restrictions imposed by the limited number of neighbors (due to regularity all interior cells have the same number of neighbors) the classical pyramid linking may also destroy the connectivity of the receptive fields [11]. Nacken's modifications not only preserve the connectivity of the receptive fields, but they also extend the original linking concept: links may move to any neighbor of a (newly) chosen parent even if it is not a neighbor of its original parent. As a consequence the number of neighbors of a cell may grow higher than in the initial state, and also the receptive fields can grow beyond the borders in the regular pyramid. The new concept of contraction kernels allows different factors of contraction at different image regions. The following proposition proves that all possible segmentations (as defined in [12]) can be represented using contraction kernels. Note that any homogeneity predicate can be used to define the segmentation $\bigcup_{i=1}^{n} R_i$.

Proposition 1 *Let* $\bigcup_{i=1}^{n} R_i = V_0, R_i \cap R_j \neq \emptyset$ *be a partition of the vertex set into connected regions* R_i. *Then there exists a dual irregular pyramid* $((G_0, \overline{G_0}), (G_1, \overline{G_1}), \ldots, (G_k, \overline{G_k}))$ *built by dual graph contraction such that*

1. *All vertices* $v_k \in V_k$ *in the top level appear in exactly one region* R_i.

2. $card(V_k) = n$.

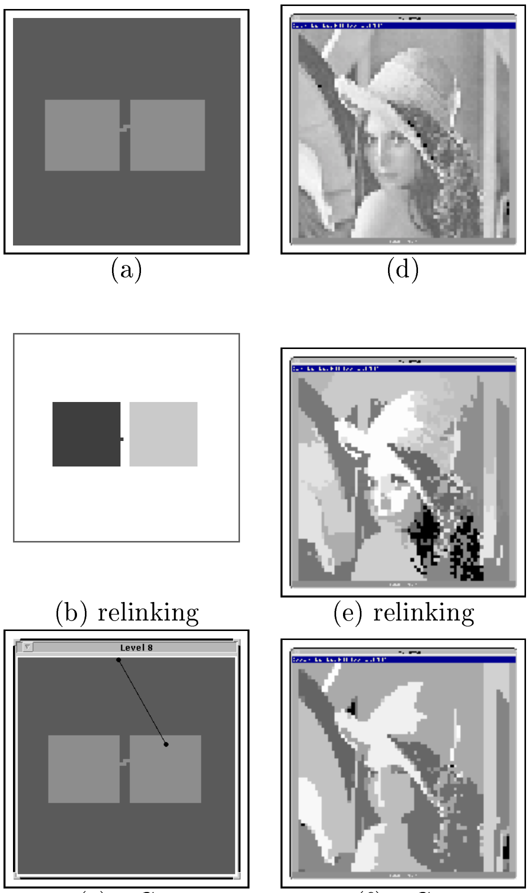
3. $card(R_i \cap V_k) = 1$ *for all regions* R_i.

4. *Let* $v_i \in R_i \cap V_k$ *and* $v_j \in R_j \cap V_k$, $i \neq j$; *then* $(v_i, v_j) \in E_k \Leftrightarrow R_i$ *and* R_j *are adjacent.*

The detailed proof is presented in [6].

4 Results

In this section we present some experimental results. The first example Fig 3(a) was taken from [1]. This example shows how classical pyramid segmentation algorithms are sensitive to shift-variance. Figure 3(b) shows the output of the segmentation processed by the classical relinking algorithm 3(b) and our algorithm 3(c). As pointed out by [1] and [11] the connectivity is not preserved with the classical relinking algorithm. The output of our algorithm preserves the connectivity. The result is shown with 2 surviving cells defining the 2 segmented regions.
Figure 4 shows the shift-variance problem of the classical relinking algorithm. The original image (Figure 3(a)) was shifted by 1 pixel in the horizontal direction and the result is Figure 4(a). In the other example Figure 4(b) the image was shifted in both vertical and horizontal direction. As it was expected there is no stability for the segmentation result when using the classical relinking algorithm with a rigid structure. Our algorithm gives exactly the same output as on Figure 3 showing hence its stability.
We applied in Fig 5, Fig 3(d), also the segmentation process of our algorithm on real data images. The results shown are satisfactory.

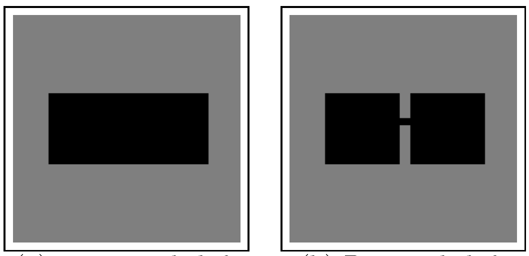


(a) (d)

(b) relinking (e) relinking

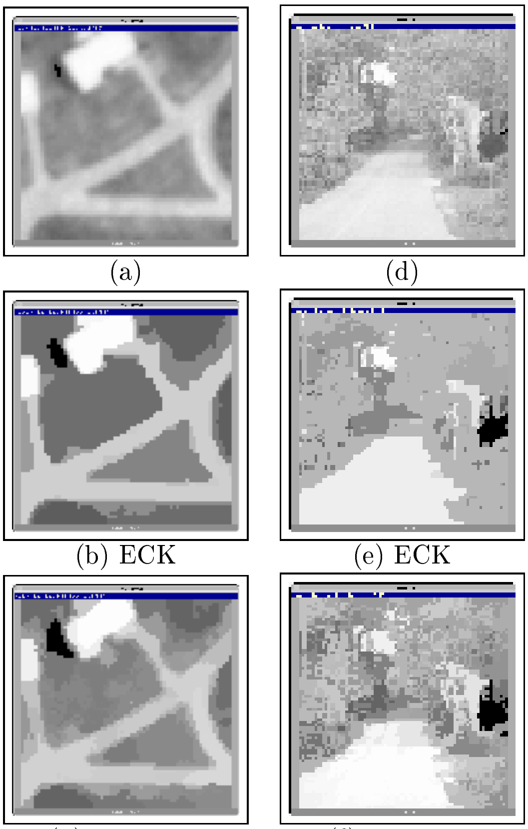
(c) ECK (f) ECK

Figure 3. Segmentation produced by "relinking algorithm" and "ECK algorithm".



(a) Horizontal shift (b) Diagonal shift

Figure 4. Shift variance segmentation of the "pyramid relinking algorithm".



(a) (d)

(b) ECK (e) ECK

(d) Relinking (f) Relinking

Figure 5. Original image and segmented images.

5 Conclusion

The proposed algorithm overcomes the problems of classical pyramid segmentation algorithms cited by [1] within a unified framework in which we give the formal proof that irregular (graph) pyramids can be used for general purpose image segmentation. The method reconstructs the segmentation in a single step using the Equivalent Contraction Kernel principle. The method is computationally [13] efficient and is highly parallel.

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