Parallel Relinking of Graph Pyramids [1)]

R. Glantz and W. G. Kropatsch

Pattern Recognition and Image Processing Group
Vienna University of Technology
Favoritenstr. 9, 183/2, A-1040 Vienna
Austria
Phone: ++43-(0)1-58801-18358
Fax: ++43-(0)1-58801-18392

e-mail: {glz,krw}@prip.tuwien.ac.at

Abstract:
In this paper we propose a new method to relink graph pyramids. The conventional method for relinking graph pyramids requires connectivity checks for the modified receptive fields. As a consequence, the relinking cannot be performed in parallel. By means of a new representation for graph pyramids, we overcome this drawback. The results are demonstrated on the task of relinking a graph pyramid such that it has a given receptive field.

1 Introduction

The task of segmenting an image and that of estimating properties of image regions may be interdependent [BHR81]. Indeed, the computation of the properties depends, in general, upon the results of segmentation. In their seminal paper about this problem Burt et al. [BHR81] suggested to recompute segmentation and image properties in an iterative parallel way. A regular image pyramid [KLB99] with variable father-son links provides the framework for the iterations. However, regular image pyramids have a major drawback: Image segmentation algorithms based on regular image pyramids are not shift invariant, i.e. slightly shifting the image may influence the segmentation considerably [BCR90]. Irregular image pyramids or *graph pyramids* are more robust in this respect [Bis95]. The levels of a graph pyramid consist of graphs, the graph on level $l+1$ being a generalization of the graph on level l. As in regular image pyramids, the image is stored in the base level. In [Nac95] the concept of variable father-son links was transferred from regular image pyramids to graph pyramids. Overcoming shift invariance, a new problem occurs: The proposed relinking method cannot be performed in parallel. In this paper we present a new representation for graph pyramids which allows the relinking to be performed in a parallel way. The paper is organized as follows. Section 2

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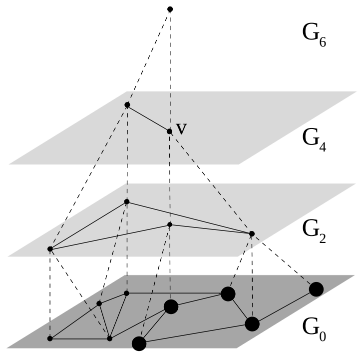

Figure 1: The vertices forming the receptive field of v are enlarged.

reviews the essentials of dual graph contraction. Section 3 points out the problems related to relinking graph pyramids in parallel. In Section 4 we demonstrate how to represent the construction of graph pyramids in the base level of the pyramid. In Section 5 we derive the parallel relinking method from the new representation. An example is given in Section 6.

2 Dual Graph Contraction

The construction of graph pyramids by *dual graph contraction* is described in [Kro95a]. Let $G_0 = (V_0, E_0)$ and $\overline{G_0} = (\overline{V_0}, \overline{E_0})$ denote a pair of plane graphs, where $\overline{G_0}$ is the dual of G_0. Dual graph contraction consists of two steps: *dual edge contraction* and *dual face contraction*. The dual edge contraction is specified by a spanning forest F_0 of E_0, the trees of which are referred to as *contraction kernels*. Each contraction kernel T_0 of F_0 is contracted to one vertex v_1 of the graph $G_1 = (V_1, E_1)$ on the next level of the graph pyramid. For each vertex v_0 of T_0 the vertex v_1 is called *father* of v_0 and v_0 is called the *son* of v_1. Each edge of E_1 corresponds to exactly one edge in E_0, which does not belong to a contraction kernel. Let $\overline{F_0}$ denote the set of edges in $\overline{E}_0$, which are dual to the edges in F_0. Set $\overline{E_1} := \overline{E_0} \setminus \overline{F_0}$ and $\overline{G_1} := (\overline{V_0}, \overline{E_1})$. Note that G_1 and $\overline{G_1}$ form a dual pair of plane graphs.

The second step, called *dual face contraction*, is specified by contraction kernels $\overline{F_1}$ in $\overline{G_1}$. Analogous to dual edge contraction, we generate $\overline{G_2}$ and set $G_2 := (V_1, E_2)$ with $E_2 := E_1 \setminus F_1$. Each vertex in G_2 has exactly one son in G_1, i.e. the vertex itself. The graphs G_2 and $\overline{G_2}$ form another dual pair of plane graphs.

Each vertex in the graph pyramid represents a connected set of base level vertices, the so called *receptive field*. The receptive field of a base level vertex contains exactly the vertex itself. For each vertex v_k on the level $k \geq 1$ the *receptive field* $RF(v_k)$ is defined by all vertices in the base level of the pyramid which lead to v_k by climbing the pyramid from sons to fathers:

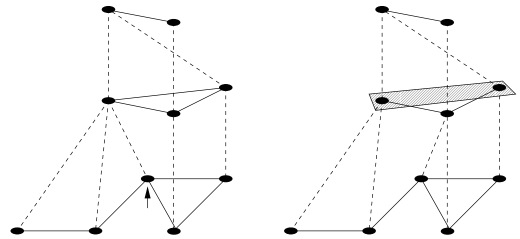

Figure 2: Relinking yields loss of connectivity in a higher pyramid level (adapted from [Nac95]).

$RF(v_0) = \{v_0\}$ for $v_0 \in V_0$,
$RF(v_k) = \bigcup(RF(v_{k-1}) \mid v_{k-1}$ is son of $v_k), \quad k > 0$.

In Fig. 1 the odd levels of the graph pyramid are omitted.

3 Modifying Father-Son Links

In the relinking of graph pyramids [Nac95] uncles can adopt sons. If levels higher than the level of the fathers are present in the graph pyramid, the receptive fields of vertices above the level of the fathers may become disconnected (Fig. 2). Complex algorithms are required to check whether multiple relinking operations may be performed in parallel. This is due to the fact, that the compatibility of relinking operations cannot be described locally [Nac95].

4 A new Representation of Graph Pyramids

Let G_0 and $\overline{G_0}$ denote a pair of plane graphs and assume $\mathcal{P} = (G_0, G_1, \ldots, G_{2n})$ and $\overline{\mathcal{P}} = (\overline{G_0}, \overline{G_1}, \ldots, \overline{G_{2n}})$ to be graph pyramids constructed on top of the pair $(G_0, \overline{G_0})$ by dual graph contractions. We also assume that the apex G_{2n} is a graph with one vertex and no edges. Let $G_i = (V_i, E_i)$ for all $0 \leq i \leq 2n$. For each edge $e \in E_0$ there exists a maximal level $l(e)$ of $\mathcal{P}$ which contains e, i.e.

$$l(e) := max\{j \mid e \in E_j \setminus E_{j+1}\}. \tag{1}$$

In [Kro95b] it is shown that the set of edges, which are contracted by dual edge contraction, i.e. the set

$$SP = SP(\mathcal{P}) := \{e \in E_0 \mid l(e) \equiv 0 \bmod 2\} \tag{2}$$

forms a spanning tree of G_0. Conversely, let $l(\cdot)$ denote a function from E_0 into the set of integers, such that

- the edges with an even l-value form a spanning tree of G_0 and
- the edges dual to the edges with an odd l-value form a spanning tree of $\overline{G_0}$.

The function $l(\cdot)$ specifies a pair of graph pyramids, where the contraction kernels on level k above G_0 [$\overline{G_0}$] are given by the [dual of] the edges with l-value equal to even [odd] k.

5 Parallel Relinking

Let the construction of a pair $\mathcal{P} = (G_0, G_1, \ldots, G_{2n})$ and $\overline{\mathcal{P}} = (\overline{G_0}, \overline{G_1}, \ldots, \overline{G_{2n}})$ of graph pyramids be specified by a function $l(\cdot)$ as explained in Section 4. The goal of the relinking is formulated by the assignment of *favored* l-values $l_{fav}(\cdot)$ to a subset E_{sel} of edges from E_0.

Let E_{sel}^{even} and E_{sel}^{odd} denote the set of edges in E_{sel}, whose favored l-values are even and odd respectively.

The parallel relinking is performed by constructing new graph pyramids on top of $(G_0, \overline{G_0})$ in the following way:
For $j = 0$ to $2n - 1$ do

- As long as there are non-self loops $e(j)$ [$\overline{e}(j)$] in G_j [$\overline{G_j}$] with e_j in E_{sel}^{even} [E_{sel}^{odd}] and $l_{fav}(e_j) = j$: Generate contraction kernels from these edges in G_j [$\overline{G_j}$] and dually contract.

- As long as there are non-self loops $e(j)$ [$\overline{e}(j)$]: Generate contraction kernels from these edges in G_j [$\overline{G_j}$] and dually contract.

- $j := j + 1$;

If $G_{2n} \neq \emptyset$, perform dual graph contraction until there are no more edges.

The relinking algorithm maximizes the number of edges with a favored l-value. To see this consider the following weight functions:

$$w_0(e) := \begin{cases} -1 & : \quad e \in E_{sel}^{even} \\ 0 & : \quad e \in (E_0 \setminus E_{sel}^{odd}) \setminus E_{sel}^{even} \end{cases}$$

and

$$w_1(e) := \begin{cases} -1 & : \quad e \in E_{sel}^{odd} \\ 0 & : \quad e \in (E_0 \setminus E_{sel}^{even}) \setminus E_{sel}^{odd} \end{cases}$$

and note that

- E_{sel}^{even} and E_{sel}^{odd} are disjoint and

- the relinking algorithm yields a minimal spanning forest with respect to $w_0(\cdot)$ and a minimal spanning forest with respect to $w_1(\cdot)$.

6 Example

Consider a hierarchy of receptive fields as described by a graph pyramid $\mathcal{P}$ on top of G_0. An example is given in Fig. 3. In this section the following question is solved: How to relink the pyramid in a parallel way such that one of its receptive fields equals a given connected set S of vertices from G_0? In any case, $\mathcal{P}$ has a unique smallest receptive field $RF_{cov} = RF(v_{cov})$ that covers S. Let the even pyramid level which contains v_{cov} be denoted by l_{cov}.

As proposed in Section 5, the goal of the relinking is formulated by the assignment of favored l-values $l_{fav}(\cdot)$ to a subset E_{sel} of edges from E_0. Let again E_{sel}^{even} and E_{sel}^{odd} denote the set of edges in E_{sel}, whose favored l-values are even and odd respectively.

In the relinked pyramid the set S_{con} of edges that connect a vertex in S with a vertex not in S will have the following property: All but one edge have a label equal to 1 and the only even label equals l_{cov}. An element e_{con} is chosen from E_{con} by random. Set

$$l_{fav}(e)) := \begin{cases} l_{cov} & : \quad e = e_{con} \\ 1 & : \quad e \in E_{con} \setminus \{e_{con}\} \end{cases}$$

In order to assure that S is connected by edges whose l-value is smaller than l_{cov}, we set

$$l_{fav}(e)) := 1 \qquad \text{if } e = (u, v) \text{ with } u, v \in S, \text{ and } l(e) \equiv 1 \bmod 2.$$

7 Conclusion

The new representation of graph pyramids allows to relink graph pyramids in a parallel way. The new relinking method can be applied to tracking, motion analysis and graph based object recognition.

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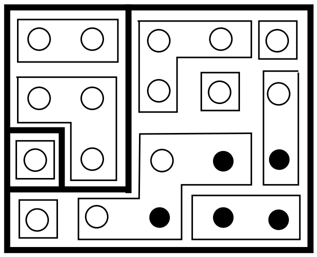
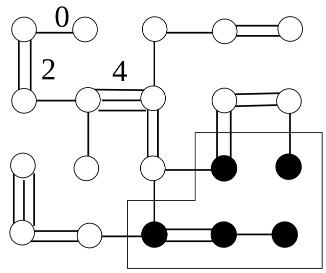
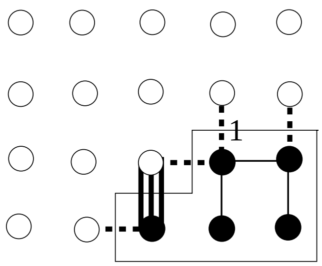


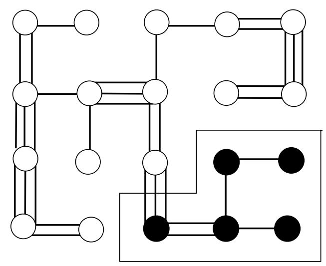
(a) Receptive fields before relinking.



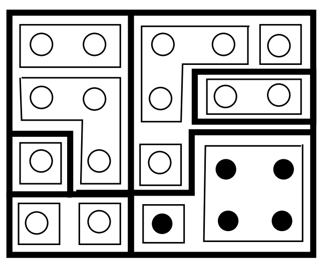
(b) Initial even labels



(c) Favored labels



(d) Even labels after relinking



(e) Final receptive fields

Figure 3: Relinking towards a given receptive field (set of filled circles).

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