The optimal height of a graph pyramid[1)]

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Abstract:
In many applications and especially in remote sensing often huge homogeneous regions, like lakes, are to be represented by a single entity. The question is how can a region be contracted in an optimal way while preserving topological properties and relations to the surrounding? We present a deterministic algorithm which first finds a minimal spanning tree of the graph corresponding to one region in the image. After that, the diameter and the center is found. Then it recursively decomposes the minimal spanning tree into subtrees until all subtrees have almost a depth of one. Each decomposition corresponds to one contraction kernel that can be used for topology preserving dual graph contraction.

1 Introduction

Many processes on regular pyramids [4] have a parallel computational complexity that depends on the number of levels of the pyramid. Such processes work on all elements of a level simultaneously and propagate information between levels either bottom-up or top-down or they combine both directions. The number of levels of a regular pyramid is determined by a constant reduction factor [7]. It is 4 for a pyramid with levels of $2^i \times 2^i, 2^{i-1} \times 2^{i-1}, \ldots, 2^0 \times 2^0$ pixels. The reduction factor determines the number of cells by which the reduced level shrinks after each reduction. Also, the reduction factor can be expressed alternatively by the diameter L of the base level. The diameter of a connected set of nodes is defined as follows: Let $\text{dist}(u, v)$ denote the length of the shortest path between two cells u and v in terms of steps between nodes. Then the diameter is the maximum of all $\text{dist}(u, v)$ among all different pairs of cells u, v. If 4-connectivity is used, the image diameter of a rectangular $n \times m$ array is $n + m$. A reduction factor f typically reduces the size of the array to $n \times m/f = n/\sqrt{f} \times m/\sqrt{f}$

[1)] This paper has been supported by the Austrian Science Fund (FWF) under grant P14445-MAT and P14662-INF.

with a diameter of $L = (n+m)/\sqrt{f}$. Graph pyramids have a greater flexibility in allowing certain parts to shrink faster than others. Some subparts may even keep their size waiting for their surrounding to provide the necessary information to continue shrinking. Therefore it is not evident that graph pyramids have a similar computational complexity as regular image pyramids. Reduction in graph pyramids is a contraction process which is controlled by contraction kernels. Contraction kernels are subtrees of the graph that contract into a single vertex of the next higher pyramid level. Several successive contractions can be combined into a single contraction which is controlled by an equivalent contraction kernel (ECK)[5] which is a tree spanning the receptive field of the surviving vertex. This property allows us not only to achieve faster contraction rates by larger contraction kernels, but also to decompose large contraction kernels into smaller ones in a globally efficient way if no external constraint imposes a specific decomposition. This is the key idea for the present paper. In images we often see large homogeneous regions without any specific substructure. These regions need to be shrinked into a single vertex of the region adjacency graph. In such case it is important to summarize the properties of the large region in a small number of steps. In terms of graph contraction we search for a decomposition of a tree spanning the connected region into a number of local contraction kernels which can shrink the large region in a few parallel contraction steps into a single vertex. For this purpose each contraction kernel should form a maximum independent vertex set (MIS) like Meer's stochastic pyramid in [7]: The kernels span the whole region and consist of roots and leafs only, and roots are not allowed to be adjacent. This splits the problem into two:

1. Find a minimum spanning tree τ of the region.
2. Decompose τ into a few local MIS spanning forests, which contract the region by a few successive steps.

The first problem can be solved by several classical algorithms [6]. For the second problem we propose the following algorithm called Recursive Decomposition of a Tree (RDT) which aims at an efficient decomposition of a given τ:

1. Determine the diameter L and the 'center' of τ.
2. Decompose τ, into subtrees τ_i with diameter not greater than $L/2$.
3. Recurse steps 1 and 2 on the subtrees τ_i until their diameters $L_i \leq 2$ for all subtrees. This recursive decomposition actually stops after $\log(L)$ steps, since the maximum diameter at iteration k is $L/2^k$.

2 Motivations

Given a graph as base level of the pyramid, a minimal spanning tree τ is built and its diameter L is found. The diameter directly relates to the optimal height h of the pyramid, if contraction kernels are restricted to depth one: $h = \log(L)$. Hence, the optimal height of the pyramid is known in advance.

In [2, 1] contraction kernels are selected stochastically. In this paper, they are constructed in a deterministic way: Using a recursive decomposition of τ into subtrees, contraction kernels of depth one are built.

From an apex of the pyramid with optimal height we can make down projection of the apex to the base level in optimal time.

The RDT algorithm presented in section 1 allows for contraction kernels of depth one and also higher depth, which enables us to construct pyramids of any height $h \geq 2$.

3 Experiments and Results

To find an optimal pyramid deterministically, the RDT algorithm is applied which gives us an optimal height of a pyramid. Table 1 compares the height h of pyramids constructed by the RDT algorithm with three stochastic methods: MIS (Maximal Independent Set) [7], MIES (Maximal Independent Edge Set), and MIDES (Maximal Independent Directed Edge Set) [1]. Each graph was randomized by stochastically contracting it to about 4000 nodes and 27700 edges. Following values are compared: The minimum height $min(h)$ of the pyramids, the maximum height $max(h)$, the average height $\overline{h}$, and the variance $\sigma(h)$. All methods are applied to the same graphs. All 100 pyramids constructed with RDT, have a lower height. A particular decomposition is shown in Figure 1.

Algorithm	$min(h)$	$max(h)$	$\overline{h}$	$\sigma(h)$
Stochastic with MIS	9	20	12.39	2.4367
Stochastic with MIES	10	11	10.26	0.4408
Stochastic with MIDES	8	11	8.73	0.6172
Deterministic with RDT	7	8	7.94	0.0119

Table 1: Deterministic and stochastic contraction: comparison of the pyramids' height h

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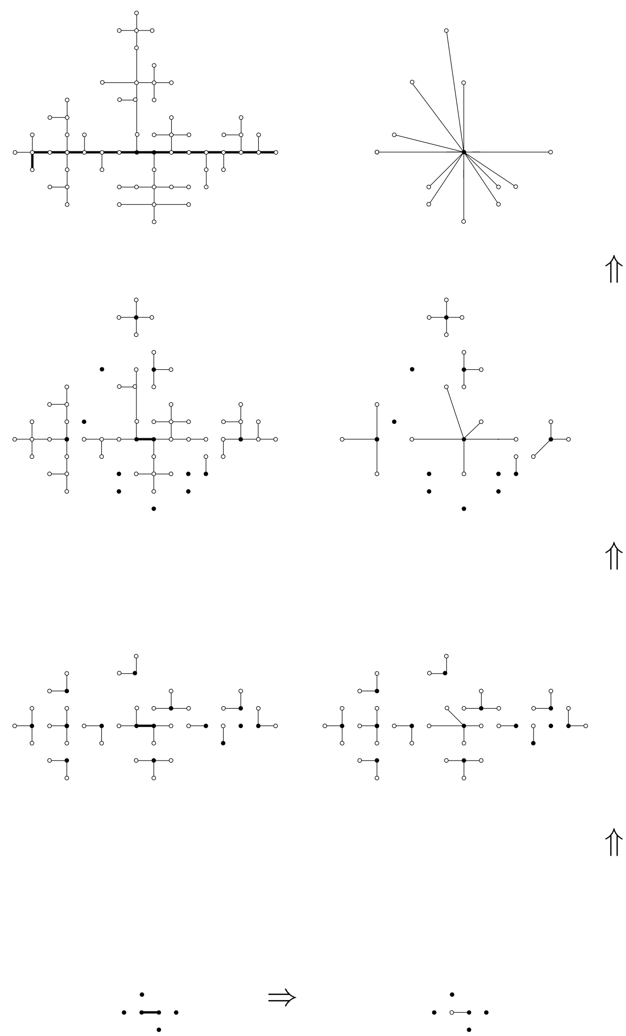

(τ3) Diameter of a tree τ3 with length 15. The center of each subtree is black.

(CK3) Contraction kernels CK3 connects the components of the tree in (τ3). Surviving nodes are black.

(τ2) first decomposition by RDT. Subtrees with diameter less than 2 are no more contracted.

(CK2) Consist of :trees od diameter no more than 2 of τ2 and of trees connecting decomposed component of τ2

(τ1) second decomposition by RDT.

(CK1)

(τ0) Final decomposition.

(CK0) Contracts the central edge only

Figure 1.

Workshop. Austria, 2002.

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Questions:

1. What is the original contribution of the work?
   Recursice Decomposition of a Tree (RDT) presents a new algorithm for constructing irregular pyramids of height log(diameter). The theoretical optimality is verified by an experimental comparison.

2. Why should this contribution be considered important?
   Irregular pyramids provide universal segmentation, the logarithmic height contributes to the computational efficiency of the 'vertical' processes on the hierarchy.

3. What is the most closely related work by others and how does this work differ?
   [7, 3] are locally optimal bottom-up constructions, while RDT provides a globally optimal hierarchical decomposition by a top-down strategy.

4. How can other researchers make use of the results of this work?
   The simple algorithm may turn out useful in nearly all applications where multi-scale representations are used.

5. Has this work been presented/submitted elsewhere?
   This is the first presentation of RDT. Irregular pyramids based on dual graph contraction have been presented before, also within the ÖAGM. That is why this basic concept is not repeated here.

6. Which form of presentation is preferred: Oral or Poster?
   Oral

7. Are you eligible for the best paper award(researcher without PhD or with the paper about the just finished thesis)?
   No