Hierarchical Image Partitioning using Combinatorial Maps

Yll Haxhimusa [a], *Adrian Ion* [a], *Walter G. Kropatsch* [a], *and Luc Brun* [b]

[1)] [a] Pattern Recognition and Image Processing Group 183/2, Vienna University of Technology

[b] Groupe de Recherche en Informatique, Université de Caen Basse

[a]{yll,ion,krw}@prip.tuwien.ac.at, [b]luc.brun@greyc.ensicaen.fr

Abstract:

We present a hierarchical partitioning of images using a pairwise similarity function on a combinatorial map based representation. We used the idea of minimal spanning tree to find region borders quickly in a bottom-up way, based on local differences. The result is a hierarchy of image partitions with multiple resolutions suitable for further goal driven analysis. The algorithm can handle large variation and gradient intensity in images. Dual graph representations lack an explicit encoding of the orientation of planes, existing in combinatorial maps.

1 Introduction

The authors in [10] suggested to bridge and not to eliminate the representational gap, and to focus efforts on *region segmentation*, *perceptual grouping*, and *image abstraction*. They employ region-adjacency-graph based technique in order to produce *meta-regions*[2)] of a particular view of a generic model. They start from a single vertex, representing a single region (silhouette), which is derived from the input region adjacency graph (produced by graph-based segmentation method in [5]), by merging in a pairwise manner the regions of an image example. After having the apexes (of two or more images) they proceed in a top-down manner to find the decomposition of each apex region into two subregions by comparing corresponding shapes and *relations* among the corresponding regions.

The union of regions forming the group is again a region with both internal and external properties and relations. Low-level cue image segmentation cannot and should not produce a complete final *good* segmentation, because there is *an intrinsic ambiguity* in the exact location of region boundaries in digital images. Problems emerge because homogeneity of low-level cues will not map to the semantics [10], and the degree of homogeneity of a region is in general quantified by threshold(s) for a given measure [6]. A *grouping method* should have the following proper-

[1)] This paper has been supported by the Austrian Science Fund under grant FSP-S9103-N04.

[2)] An output region adjacency graph as abstraction of the particular view.

ties [5]: capture perceptually important groupings or regions which reflect global aspects of the image, be highly efficient, running in time *linear* in the number of image pixels (e.g *minimal spanning tree*), and creating *hierarchical partitions* [17].

In a regular image pyramid the number of pixels at any level l, is r times higher than the number of pixels at the next reduced level $l+1$. The so called reduction factor r is greater than one and it is the same for all levels l. If s denotes the number of pixels in an image I, the number of new levels on top of I amounts to $log_r(s)$. Thus, the regular image pyramid may be an efficient structure for fast grouping and access to image objects in top-down and bottom-up processes. However, the authors in [1] conclude that regular image pyramids have to be rejected as general-purpose segmentation algorithms, because they lack shift invariance. In [15, 9, 14] it was shown how these drawbacks can be avoided by irregular image pyramids, the so called adaptive pyramids, in image segmentation and feature detection.

Region adjacency graphs (RAG), dual graphs [8] and combinatorial maps have been used before [3] to represent the partitioning of $2D$ space. From these three structures, we use the combinatorial maps because, RAGs cannot correctly encode multiple boundaries and inclusions, and dual graphs lack the explicit encoding of edge orientation around vertices (see Section 2.1 for a problem in $2D$), present in a combinatorial map [3]. Moreover with combinatorial maps, its dual must not be explicitly represented because one combinatorial map is enough to fully characterize the partition, and its dual can be easily deduced anytime.

In this paper we present a hierarchical method, introduced in [8], which in a bottom-up method produce a stack of region adjacency combinatorial maps (called *irregular combinatorial pyramid* [3]), and at the same time preserves the proper *topology* among regions during the merging processes, suitable for the top-down decomposition (e.g. as in [10]). Combinatorial maps and combinatorial pyramids are shortly presented in Section 2. Borůvka's algorithm [2] was used to build a minimal spanning tree (MST), since it was easily integrated in our combinatorial pyramid concept. Combinatorial map contraction is presented in Section 3. In Section 4 we present some experimental results.

2 Combinatorial Maps

We refer to following definitions and properties of combinatorial maps $G = (\boldsymbol{\mathcal{D}}, \sigma, \alpha)$, see [4]: set of darts $\boldsymbol{\mathcal{D}}$, permutations σ and $\alpha(d)$[4] (see Figure 1a), self-loops, bridges, dart partition,

[3] called lattice in [10].

[4] Orbits of σ encode the vertices and that of α edges, if the darts are encoded by positive and negative integers, α can be implicitly encoded by $\alpha(d) = -d$.

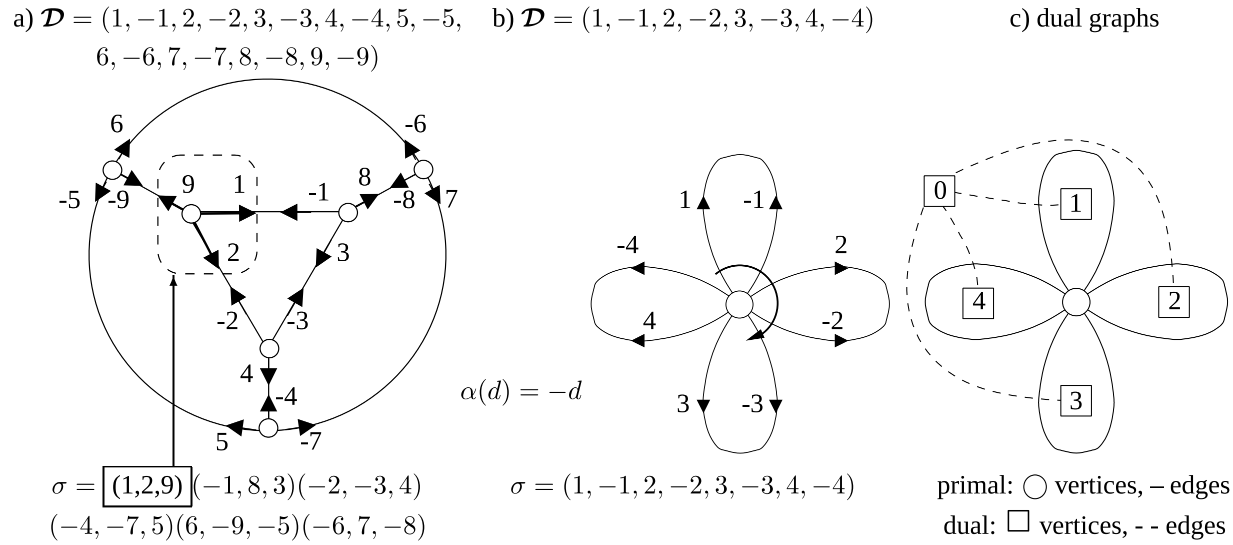

a) $\boldsymbol{\mathcal{D}} = (1, -1, 2, -2, 3, -3, 4, -4, 5, -5, 6, -6, 7, -7, 8, -8, 9, -9)$
b) $\boldsymbol{\mathcal{D}} = (1, -1, 2, -2, 3, -3, 4, -4)$
c) dual graphs
6
-6
-5
-9
9
1
-1
8
-8
7
2
3
-2
-3
4
-4
5
-7
$\sigma = \boxed{(1,2,9)}(-1, 8, 3)(-2, -3, 4)$
$(-4, -7, 5)(6, -9, -5)(-6, 7, -8)$
1
-1
-4
2
4
-2
3
-3
$\alpha(d) = -d$
$\sigma = (1, -1, 2, -2, 3, -3, 4, -4)$
0
1
4
2
3
primal: ○ vertices, – edges
dual: □ vertices, - - edges

Figure 1: a), b) Combinatorial maps. b), c) Topological case handled correctly only by combinatorial map.

contraction of a dart $G/\alpha^*(d) = \overline{\overline{G} \setminus \alpha^*(d)}$, and contraction kernel $K = \{\boldsymbol{\mathcal{D}}_1, ..., \boldsymbol{\mathcal{D}}_n\}$. A combinatorial map may be seen as a planar graph encoding explicitly the orientation of edges around a given vertex. Thus all graph definitions used in irregular pyramids [11] are analogously defined. A *combinatorial pyramid* is a stack of successively reduced combinatorial maps by the set of contraction and removal operations, i.e. $(G_0, ..., G_k)$, where k represent the levels of the pyramid. Each map $k + 1$ is build from the one below, k, by selecting a set of contraction kernels $K_{k,k+1}$ and applying it to a given combinatorial map G_k to get the reduced $G_{k+1} = \mathcal{C}[G_k, K_{k,k+1}] = G_k \setminus K_{k,k+1}$. More on removal of the redundant edges can be found in [3].

2.1 Combinatorial Maps versus Dual Graphs

Advantages of combinatorial maps over dual graphs come form the embedding, that is inherently present at the former ones. Let us analyze the *'flower'* example given in Figure 1b,c w.r.t uniqueness of topological representation. The combinatorial map of this *'flower'* is shown and defined in Figure 1b) by $G = (\boldsymbol{\mathcal{D}}, \sigma, \alpha)$[5]. If the leafs of the 'flower' exchange position for e.g. leafs 1 and 3, a different $\sigma = (3, -3, 2, -2, 1, -1, 4, -4)$ will be defined, hence uniquely encoding the topology. The dual graphs are encoded by a pair of graphs, the (planar) primal graph vertices) and its dual. For each edge in the primal graph there is a corresponding one in the dual, that crosses it (Figure 1c). Since there is no ordering of the edges around the vertices, the dual graph representation does not uniquely encode the topology of the *'flower'*, as can be

[5] σ is encoded clockwise, shown with the arrow in Figure 1b).

easily seen if we exchange the position, for e.g. of leafs 1 and 3, the dual graph describing this configuration is isomorph with the previous one (the one without exchanging the position of leafs 1 and 3).

3 Image Partitioning

The goal is to find partitions such that these elements satisfy certain properties. The authors in [5] defines a function, which measures the difference along the boundary of two components relative to a measure of the differences of components' internal differences. This definition tries to encapsulate the intuitive notion of contrast: a contrasted zone is a region containing two connected components whose inner differences (*internal contrast*) are less than differences within it's context (*external contrast*). We define an external contrast measure between two components and an internal contrast measure of each component, analogously to [5, 7].

Let $P_k = \{CC_i^k, CC_j^k, ..., CC_n^k\}$ be the partitions on the level k of the pyramid i.e P_k is the attributed combinatorial map $G_k(\boldsymbol{\mathcal{D}}_k, \sigma_k, \alpha_k, a_k)$, its vertex set $\sigma_k^*(D_k)$ by V_k, where $a : \boldsymbol{\mathcal{D}} \rightarrow \mathbb{R}^+$ and edge set $E = \alpha^*(\boldsymbol{\mathcal{D}})$. One way to attribute the darts is given in Section 4. Every vertex $u \in V_k$ is a representative of a component $CC_i = RF(u_i)$[6] of the partition P_k. The equivalent contraction kernel of a vertex $u \in V_k$, $K_{0,k}(u)$ is a set of darts (a subtree) of the base level $d \in \boldsymbol{\mathcal{D}}_0$ that are contracted; i.e. applying equivalent contraction kernel on the base level, one contracts the sub combinatorial map $G' \subseteq G$ onto the vertex u.

The *internal contrast* of the $CC_i \in P_k$ is the *largest dissimilarity* of the component CC_i i.e. the largest dart of the $K_{0,k}(u)$ of a vertex $u \in V_k$:

$$Int(CC_i) = max\{a(d), d \in \alpha^*(K_{0,k}(u))\}. \tag{1}$$

Let $u_i, u_j \in V_k$ be the end vertices of a dart $d \in \boldsymbol{\mathcal{D}}_k$. The *external contrast* between two components $CC_i, CC_j \in P_k$ is the *smallest dissimilarity* between component CC_i and CC_j i.e. the smallest dart weight connecting $K_{0,k}(u_i)$ and $K_{0,k}(u_j)$ of vertices $u_i \in CC_i$ and $u_j \in CC_j$:

$$Ext(CC_i, CC_j) = min\{a(d), d \in K_{0,k}(u_i) \wedge -d \in K_{0,k}(u_j)\}. \tag{2}$$

The pairwise comparison function $Comp(\cdot,\cdot)$ between two connected components is defined:

$$Comp(CC_i, CC_j) = \begin{cases} \text{True} & \text{if } Ext(CC_i, CC_j) > PInt(CC_i, CC_j), \\ \text{False} & \text{otherwise}, \end{cases} \tag{3}$$

where $PInt(CC_i, CC_j)$ is the minimum internal contrast difference between two components:

[6] connected components CC, and receptive field (RF)

$$PInt(CC_i, CC_j) = min(I(CC_i) + \tau(CC_i), I(CC_j) + \tau(CC_j)). \quad (4)$$

For the function $Comp(CC_i, CC_j)$ to be true i.e. for the border to exist, the external contrast difference must be greater than the internal contrast differences. The reason for using a threshold function $\tau(CC)$ in Equation (4) is that for small components CC, $Int(CC)$ is not a good estimate of the local characteristics of the data, in extreme case when $|CC| = 1$, $Int(CC) = 0$. Any non-negative function of a single component CC, can be used for $\tau(CC)$ [5]. One can define τ to be function of the size of CC: $\tau(CC) = \alpha/|CC|$, where $|CC|$ denotes the size of the component CC and α is a constant. More complex definition of $\tau(CC)$, which is large for certain shapes and small otherwise would produce a partitioning which prefers certain shapes. Algorithm 1 shows how to build the hierarchy of partitions.

Algorithm 1 – Construct Hierarchy of Partitions

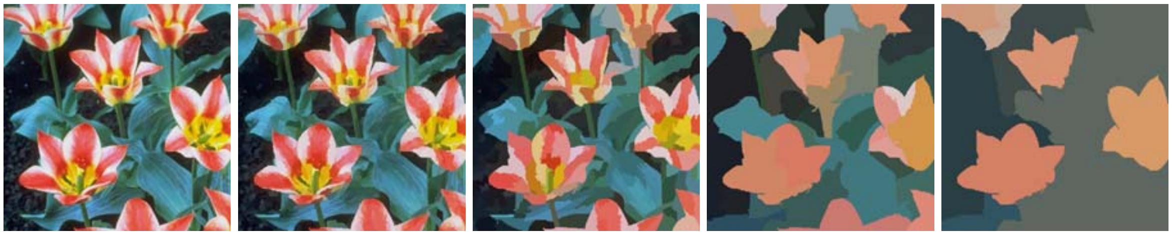
Input: Attributed combinatorial map G_0.

```
$k = 0$
repeat
    $\forall u \in V_k = \sigma_k^*(\boldsymbol{\mathcal{D}}_k)$
    $D_{min}(u) = \{d \in \boldsymbol{\mathcal{D}} | a(d) = min\{a(d')|d \in \sigma^*(d)\}\}$
    $\forall d \in D_{min}, u_i^k = \sigma_k^*(d), u_j^k = \sigma_k^*(-d)$ with $Ext(CC_i^k, CC_j^k) \leq PInt(CC_i^k, CC_j^k)$
    include $d$ and $-d$ in contraction kernel $K_{k,k+1}$
    contract combinatorial map $G_k$ with contraction kernel, $K_{k,k+1}$: $G_{k+1} = \mathcal{C}[G_k, K_{k,k+1}]$.
    set $a(d_{k+1}) = min\{a(d_k) \,|\, d_{k+1} = \mathcal{C}[d_k, K_{k,k+1}]\}$
    $k = k + 1$
until $G_k = G_{k-1}$
```

Output: An attributed combinatorial map at each level of the pyramid $(G_0, G_1, ..., G_k)$.

4 Experiments on Image Maps

All the details here (as in the whole paper) are given for the combinatorial map in which vertices represent regions. We could use also the dual combinatorial map, where faces are used to represent regions as well. One can always obtain the dual of the combinatorial map, therefor there is no need of storing both. Anyhow the same framework could be used to store and manage both representations. Because the base entity in a combinatorial map is the dart, we cannot have a map containing only one vertex and no edges. So one notable difference would be that if vertices are chosen to represent the regions, a one region map, without self loops is not possible until the background (the infinite region) is represented explicitly (darts have to be specially



(a) 0 (160 000) (b) 21 (2387) (c) 27 (321) (d) 37 (41) (e) 42 (9)

Figure 2: Some levels of the partitioning of 'Tulips': level (number of components).

added for that). On the dual, where regions are represented by faces, the background/infinite region also exists, but no special darts have to be added to accommodate it, so a one region map would be made out of two darts that are 'sewed' together by α and by σ.

We start with the trivial partition, where each pixel is a homogeneous region. The attributes of edges can be defined as the difference between end point features of end vertices, $attr_e(u_i, u_j) = |F(u_i) - F(u_j)|$, where F is some feature. F could be defined as $F(u_i) = I(u_i)$, for gray value intensity images, or $F(u_i) = [v_i, v_i \cdot s_i \cdot \sin(h_i), v_i \cdot s_i \cdot \cos(h_i)]$, for color images in HSV color distance [17]. However the choice of the definition of the weights and the features to be used is in general a hard problem, since the grouping cues could conflict each other [13].

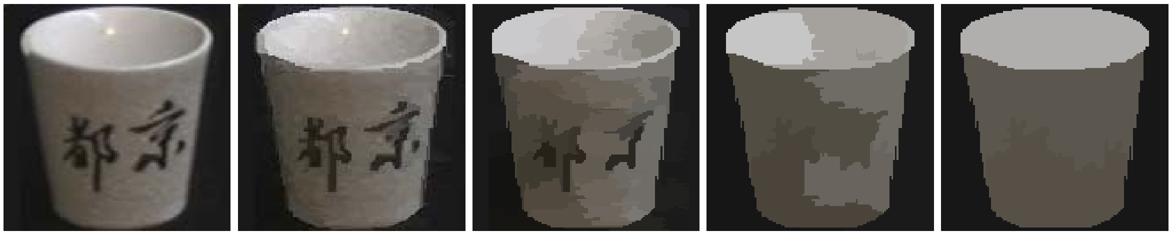
For our experiments we used as attributes of edges the euclidean distance between pixel RGB values, $a(u_i, u_j) = |rgb(u_i) - rgb(u_j)|$. We choose this simple color distance to study the properties of the algorithm. To compute the hierarchy of partitions we also need to define $\tau(CC) = \alpha/|CC|$, where $\alpha = const$ and $|CC|$ is the number of elements in CC, i.e. the size of the region. The algorithm has one running parameter α, which is used to compute the function τ. A larger constant α sets the preference for larger components. Note that as size of $|CC|$ gets larger, which happens as the algorithms proceeds toward the top of the pyramid, the function $\tau \rightarrow 0$, which means that the influence of the parameter α decreases.

We found that $\alpha = 300$ produces the best hierarchy of partitions of the images shown in *Tulips* Figure 2, *Obj18_355* Figure 3, and *Obj59_0* [7]Figure 4. Figures 2, 3 and 4 show some of the partitions on different levels of the pyramid and the number of components. In general the top of the pyramid will consist of one vertex, an apex, which represents the whole image. Note that in all images there are regions of large intensity variability and gradient. This algorithm copes with this kind of gradient and variability. In contrast to [5][8] the result is a hierarchy of partitions with multiple resolutions, suitable for further goal driven, domain specific analysis [9]. On the lower

[7] Waterloo image database and Coil 100 image database

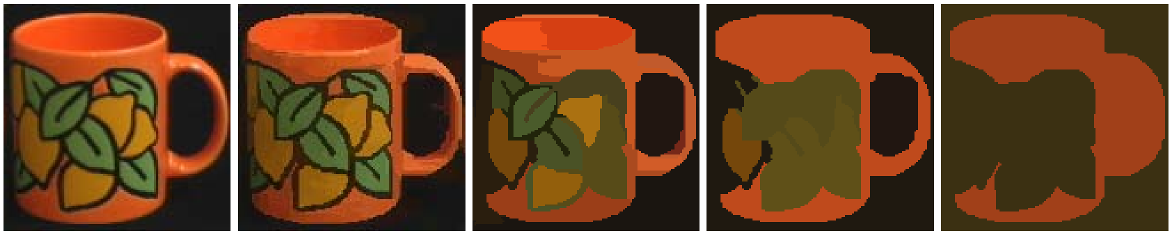
[8] In [16] results of different segmentation methods, including the ones in [5] and [13], are shown and compared.

[9] Please note that a whole class of partitions is created, where a partition is not limited to a certain level of



(a) 0 (16 384) (b) 23 (512) (c) 32 (65) (d) 39 (10) (e) 42 (4)

Figure 3: Some levels of the partitioning of 'Obj18 _355': level (number of components).



(a) 0 (16 384) (b) 24 (291) (c) 34 (23) (d) 37 (6) (e) 39 (2)

Figure 4: Some levels of the partitioning of 'Obj59 _0': level (number of components).

level of the pyramid the image is over segmented (partitioned) whereas in upper it is under segmented (partitioned), the help of mid and high level knowledge would select the proper partitioning. An approach based on some statistical measures to decide which level is the most appropriate could be used as well [12]. Since the algorithm preserves details in low-variability regions, a noisy pixel would survive through the hierarchy. Of course, image smoothing in low variability regions would overcome this problem. We, however do not smooth the images, as this would introduce another parameter into the method. The hierarchy of partitions can also be built from an over segmented image to overcome the problem of noisy pixels. The constant α is used to produce a kind of the over segmented image. For an over segmented image, where the size of regions is large, there is no need to define the function τ, thus the algorithm becomes parameterless.

5 Conclusion

In this paper we presented a method for building hierarchical image partitions using Borůvka's minimal spanning tree algorithm. The hierarchy is presented as a combinatorial pyramid, where each level is a 2D combinatorial map. Combinatorial maps are defined in any dimension, thus the current work should lead the way to segmentation of digital video streams using contraction in 3D combinatorial maps/pyramids. It was shown that the algorithm can handle large variation

the pyramid, but can be constructed of components from different levels (the receptive fields of the vertices of a multilevel partition occupy the whole image, and do not overlap) and gradient intensity in images. Even though the algorithm makes greedy decisions locally, it produces perceptually important partitions in a bottom-up way based only on local differences. A drawback is that maximum and minimum criterion is very sensitive to noise, although in practice it has a small impact. To overcome the problem of noise, one could start with an over segmented image produced by a robust method e.g. robust watershed method. A comparison between the classes of partitions produced by the presented method, and some of the well known methods (e.g. [5, 17]) is planned.

References

[1] M. Bister, J. Cornelis, and A. Rosenfeld. A critical view of pyramid segmentation algorithms. *Pattern Recognition Letters*, 11(9):605–617, September 1990.

[2] O. Borůvka. O jistém problému minimálnim. *Pr áce Mor. Přírodvěd. Spol. v Brně (Acta Societ. Scienc. Natur. Moravicae)*, 3(3):37–58, 1926.

[3] L. Brun and W. Kropatsch. Contraction kernels and combinatorial maps. *Pattern Recognition Letters*, 24(8):1051–1057, 2003.

[4] L. Brun and W. G. Kropatsch. Dual contraction of combinatorial maps. Technical Report PRIP-TR-54, Institute f. Computer Aided Automation 183/2, PRIP Group, TU Wien, Austria, 1999.

[5] P. F. Felzenszwalb and D. P. Huttenlocher. Image segmentation using local variation. In *Proceedings of IEEE Conference on Computer Vision and Pattern Recognition*, 98–104, June 1998.

[6] C.-S. Fu, W. Cho, S, and K. Essig. Hierarchical color image region segmentation for content-based image retrival system. *IEEE Transaction on Image Processing*, 9(1):156–162, 2000.

[7] L. Guigues, L. M. Herve, and J.-P. Cocquerez. The hierarchy of the cocoons of a graph and its application to image segmentation. *Pattern Recognition Letters*, 24(8):1059–1066, 2003.

[8] Y. Haxhimusa and W. G. Kropatsch. Segmentation graph hierarchies. In A. Fred, T. Caelli, R.P.W. Duin, A. Campilho, and D. de Ridder, eds., *Joint Int. Worksh. on S+SSPR*, LNCS 3138:343–351, 2004.

[9] J.-M. Jolion and A. Montanvert. The adaptive pyramid, a framework for 2D image analysis. *Computer Vision, Graphics, and Image Processing: Image Understanding*, 55(3):339–348, May 1992.

[10] Y. Keselman and S. Dickinson. Generic model abstraction from examples. *IEEE Transactions on PAMI, special issue on Syntactic and Structural Pattern Recognition,*, 2005. to appear.

[11] W. G. Kropatsch. Parallel graph contraction for dual irregular pyramids. Technical Report PRIP-TR-28, Institute f. Automation 183/2, Dept. for Pattern Recognition and Image Processing, TU Wien, Austria, 1990.

[12] S. Lallich, F. Muhlenbach, and J.-M. Jolion, A test to controll a region growing process within a hiearchical Graph. In *Pattern Recognition*, 36(10):2201-2211, 2003.

[13] J. Malik, S. Belongie, J. Shi, and T. Leung. Textons, contours and regions: Cue integration in image segmentation. In *The International Conference on Computer Vision*, pages 918–925, 1990.

[14] P. Meer, D. Mintz, A. Montanvert, and A. Rosenfeld. Consensus vision. In *AAAI-90 Workshop on Qualitative Vision*, pages 111–115, Boston, Massachusetts, USA, July 29 1990.

[15] A. Montanvert, P. Meer, and A. Rosenfeld. Hierarchical image analysis using irregular tesselations. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 13(4):307–316, April 1991.

[16] M. Pavan and M. Pelillo. Dominiant sets and hierarchical clustering. In *ICCV03*, 2003.

[17] J. Shi and J. Malik. Normalized cuts and image segmentation. In *Proceedings IEEE Conference Computer Vision and Pattern*, pages 731–737, 1997.