A Coordinate System for Articulated 2D Shape Point Correspondences*

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Abstract

A framework for mapping a polar-like coordinate system to a non-rigid shape is presented. Using a graph pyramid, a binary shape is decomposed into connected parts, based on its structure as captured by the eccentricity transform. The decomposition is used to derive domains for the angular like coordinate. A closest point search is employed to find point correspondences.

1 Introduction

Most shape matching methods output a similarity value (e.g. [4, 5, 16, 8]), some also give correspondences of the used border points/parts [14, 1, 19], but finding all point correspondences is not straightforward.

This work maps a coordinate system to an articulated shape, with the purpose of addressing the corresponding point (or a close one) in other instances of the same shape. It is motivated by observations like: ‘one might change his aspect, alter his pose, but the wristwatch is still located in the same place on the hand’.

For correspondences of all points of the shape, the task is similar to the non-rigid registration problem used in the medical image processing community [3]. Differences include the usage of gray scale information to compute the deformation vs. the usage of a binary shape and, the registration of a whole image (in this paper) connected shape vs. the registration of a (in this paper) connected shape and, the registration of a whole image (in most cases) vs. the registration of a (in this paper) connected 2D shape. In [4], a triangulation of the shape is used as a model, which could be used to find corresponding points, but an a priori known model is still needed. In the surface parametrization community [2] a coordinate system for shapes is defined, but articulation is not considered. In [9], for small variations, correspondences between points of 3D articulated shapes are found. Recently shape matching has also moved toward decomposition and part matching, e.g. [16], mainly due to occlusions, imperfect segmentation or feature detection.

2 Eccentricity and Pyramids

Eccentricity Transform [7]: Let the shape \( S \) be a closed set in \( \mathbb{R}^2 \) and \( \partial S \) be its border. A (geodesic) path \( \pi \) is the continuous mapping from the interval \([0,1]\) to \( S \). Let \( \Pi(p_1, p_2) \) be the set of all paths between two points \( p_1, p_2 \in S \). The geodesic distance \( d(p_1, p_2) \) between \( p_1, p_2 \) is defined as the length \( \lambda(\pi) \) of the shortest path \( \pi \in \Pi(p_1, p_2) \) i.e. \( d(p_1, p_2) = \min \{ \lambda(\pi(p_1, p_2)) | \pi \in \Pi \} \), where

\[
\lambda(\pi(t)) = \int_0^1 |\dot{\pi}(t)| dt, \text{ and } \pi(t) \text{ is a parametrization of the path from } p_1 = \pi(0) \text{ to } p_2 = \pi(1).
\]

The eccentricity transform of \( S \) is \( ECC(S, p) = \max \{ d(p, q) | q \in S \} \), \( \forall p \in S \) i.e. to each point \( p \) it assigns the length of the shortest geodesic path(s) to the points farthest away. The class of 4-connected discrete shapes \( S \) defined by points on a square grid \( \mathbb{Z}^2 \) are considered. Paths are contained in the area of \( \mathbb{R}^2 \) defined by the union of the support squares for the pixels of \( S \). The distance between any two pixels whose connecting segment is contained in \( S \) is computed using the \( L_2 \)-norm.

The shape bounded single source distance transform, \( DT(S, p) \), computes the geodesic distance of all points of a shape \( S \) to the point \( p \) and is the main tool used for computing \( ECC(S, p) \). \( DT(S, p) \) can be efficiently computed using discrete circles [7] or fast marching [18].

An eccentric point is a point \( c \in S \) that is farthest away from at least one other point \( p \in S \) i.e. \( \exists p \in S \text{ s.t. } ECC(S, p) = d(p, c) \). The center \( C \subseteq S \) is the set of points with the smallest eccentricity i.e. \( c \in C \text{ iff } ECC(S, c) = \min \{ ECC(S, p) | \forall p \in S \} \). If \( S \) is simply connected, \( C \) is a single point. Otherwise it can be a disconnected set of arbitrary size (e.g. for \( S = \)

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1 It can be generalized to any continuous and discrete space.
the points on a circle, all points are eccentric and they all make up the center). The smallest eccentricity is the 
radius of the shape, and the highest one is the diameter.

Due to using geodesic distances, the variation of ECC is bounded under articulated deformation to the width of the 'joints' [14]. The transform is robust with respect to Salt & Pepper noise, and the positions of eccentric points and center are stable [13]. Fig. 1 shows two hand shapes (taken from the Kimia99 database [17]) and their eccentric transform.

**Irregular Graph Pyramids:** A graph pyramid $P = \{G_0, \ldots, G_t\}$ is a stack of successively reduced graphs. Each level $G_k = (V_k, E_k)$ is obtained by contracting and removing edges in the level below. Successive levels reduce the size of the data by $\lambda > 1$. The reduction window relates a vertex at a level $G_k$ with a set of vertices in the level directly below ($G_{k-1}$). Higher level descriptions are related to the original input data - the receptive field (RF) of a given vertex $v \in G_k$ aggregates all vertices in $G_0$ of which $v$ is an ancestor.

Each level represents a partition of the base level into connected subgraphs i.e. connected subsets of pixels in our case. The construction of an irregular pyramid is iteratively local [15]. In $G_0$ the vertices represent single pixels. The union of neighboring vertices on level $k-1$ (children) to a vertex on level $k$ (parent) is controlled by trees called contraction kernels (CK) [11] chosen by the algorithm (e.g. segmentation, connected component labeling, etc.). Every vertex computes its values independently of other vertices on the same level. Thus local independent (and parallel) processes propagate information up and down and laterally in the pyramid [12].

### 3 ECC Isoheight Lines - Decomposition

The level set [20] of a function $f : \mathbb{R}^n \to \mathbb{R}$, corresponding to a value $h$, is the set of points $p \in \mathbb{R}^n$ s.t. $f(p) = h$. A level set of the ECC of $S$ is the set $LS(e) = \{q \in S \mid ECC(S, q) = e\}$, with $e \in [\min \{ECC(S, p)\}, \max \{ECC(S, p)\}]$. For $S \in \mathbb{R}^2$, $LS(e)$ can be a closed curve or a set of disconnected open curves. The connected components of $LS(e)$ are called isoheight lines, $IL \subseteq LS(e)$, $IL$ connected.

$HD(S) = \{R_1, \ldots, R_n\}$ is a a decomposition of $S$ based on the connectivity of the ECC isoheight lines (Fig. 2) if: $HD$ is a partition of $S$ into simply connected regions; $\forall R_i \in S$ simply connected regions; $\forall e \in [\min \{ECC(S, p)\}, \max \{ECC(S, p)\}] \Rightarrow R_i \cap LS(e)$ is connected; the number $n$ of regions is minimal. $HD(S)$ exists for any connected shape $S$.

The top level $G_1$ of the graph pyramid created by Alg. 1 is a region adjacency graph describing the topology of the decomposition $HD(S)$. Edges of $G_1$ are oriented from regions with lower eccentricity to regions with higher eccentricity. Each vertex contains the length of the longest isoheight line in its RF.

If $S$ is simply connected, the obtained region adjacency graph is a tree (Theorem 7.9 in [10]), with the RF of the root vertex containing the (unique) center pixel. Such a decomposition can be done for other transforms also (e.g. the DT($S, p$)). The eccentricity transform is used because its center is a robust starting point [13]. A study of the decomposition of shapes based on ECC isoheight lines in the context of shape matching is planned.

### 4 The Non-rigid Coordinate System

A system of curvilinear coordinates [20] is composed of intersecting surfaces. If all intersections are...
is a linear mapping from the eccentricity value and the radial coordinate intuitively so. The required. Note that with cycles, etc.) and more complex algorithms are reconnected shapes is much more complex (general graph consideration the possible deformations would be optimal.

Fig. 2 shows the results of Alg. 1 and 2, and Eq. 1 and 2 for the two hands. The jagged isoheight lines of \( \theta \) are due to the smoothness/roughness of the shape boundary i.e. curvature of the shape boundary at the endpoints of isoheight lines, and partly due to the simple implementation (point projection by closest point search and integral along line estimation by sum of line segment lengths for Eq. 2, etc.).

5 Experiments

A pattern was laid on each hand - the source, and copied to the other one - the destination, by finding for each pixel \( p_d(r_d, \theta_d) \) of the destination the “closest” pixel \( p_s(r_s, \theta_s) \) in the source. The local variation of \( \theta \) is not constant over the whole shape, making the Euclidean metric not the best option for finding the closest pixel to a given point \( p_d(r_d, \theta_d) \). To avoid compensating for this variation, a two step approach is used. First, normalize \( r \) in both shapes to \([0, 1]\). This makes finding \( ecc_s \rightarrow 0 \rightarrow ecc_s \) a linear scaling problem. \( L \leftarrow (ecc_s \leq ECC(source) \leq ecc_s + 1) \) gives at least 8 connected isoheight lines of \( r \). Second, the pixel of \( L \) which minimizes \( |\theta_d - \theta_s| \) is chosen. The results are promising (see Fig. 2) with the texture of the “articulated” finger being nicely copied from one shape to the other i.e. points are copied to their corresponding region in the articulated version of the shape.

The noise like errors on the pattern are due to the approximations mentioned above and to using “nearest point” for finding the color of each pixel when copying the pattern (instead of interpolating gray values). Errors on the boundaries of fingers are due to certain coordinates not existing in both shapes. The more global perturbation (palm of the hands in Figure 2) is mainly due to the slightly different position of the centers and isoheight line shape. Improvements can be made by considering both shapes when mapping the coordinates to them, or by a more complex method for finding corresponding points. Finding a matching between the regions of the decomposition of the two shapes is an important step planned for the future.
6 Conclusion and Outlook

This paper presents a framework for mapping a polar-like coordinate system to a non-rigid binary shape and finding corresponding points between two shapes. Promising initial results are presented. More global decisions will provide smoother angular isohigh lines, and additional correspondences between part structures can help to solve failed correspondences. Further quantitative evaluation and extension to non simply connected shapes is planned.

References