Object Classification by Topology of Convex Deficiencies*

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Abstract Topology can be used to characterize the structure of objects invariant to changes in their geometry. Nevertheless there are cases in which the number and configuration of the holes is not enough to discriminate between objects of different classes. The difference between an object and its convex hull can be considered in addition. We propose to use the topology of the difference in the context of classification. This information can be computed in the pyramid framework which has already been used to efficiently compute topological invariants. Using also the topology of the difference, seam to offer a better descriptive power than Betti numbers alone. A proof of concept example, for the set of 26 capital letters of the latin alphabet shows the enhanced descriptive capabilities.

Keywords convex hull; topological invariants; irregular pyramid;

1 Introduction

A classical task in image processing, classification, associates individual items to groups called *classes*. The association is done based on information on one or more characteristics of the item to classify and of the classes. Classification requires to discriminate between items belonging to different classes (e.g. the letters A, B, and O). This is usually done by selecting and extracting properties[1] of the items, that have a small variation, preferably zero, for items in the same class, but differ a lot between two items that belong to different classes.

Topological features ignore changes in geometry caused by elastic transformations. Thus one can say that topology is robust with respect to changes in the geometry that do not affect the structure. Simple features are for example the number of connected components, the number of holes, etc., while more refined ones, like homology and cohomology, characterize holes and their relations.

Certain classification tasks (e.g. recognition of letters) cannot be solved using purely the topology of the object (e.g. both 'A' and 'R' have one single hole). In addition, connected components, tunnels and cavities do not always allow to distinguish many of the 3D objects appearing in 2D images: cavities are invisible, tunnels can be discriminated from pairs of "deep" concavities only in special cases, and some are also invisible. However every tunnel creates intentations similar to concave parts of the object's surface. Such concave surface patches can be partly invisible and create occlusion boundaries inside the silhouette of the object. Such occlusions are detected when the object moves (= self-occlusion). The only surfaces where such self-occlusions cannot happen are convex objects.

In [2] the idea of looking at the properties of the difference between an object and its convex hull has been presented. The connected components of the difference are called *convex deficiencies*

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[1] also called features.

(CD). The distance transform and a 3D version of the watershed algorithm are used to discriminate concavities from tunnels, and skeletons are used to describe the obtained CDs. As an open problem, "a better representation" for the obtained CDs is mentioned. The usage of these CDs in the context of classification is not approached.

This paper considers the difference between an object and its convex hull (CDs), in the context of classification for rigid objects. The topology of the difference is proposed as a feature for classification in addition to the topology of the object. It can be described with different topological invariants (e.g. Euler characteristic, Betti numbers). The topology of the object and of the difference can be computed efficiently using a graph pyramid [14].

The paper is organized as follows. Section 2 recalls some of the known topological invariants and shortly discusses their properties. Section 3 briefly recalls the concept of convex hull commonly used and discusses the topology of the difference, its computation and illustrates it in the context of a simple classification example. Section 4 concludes the paper.

2 Topological Invariants

A topological invariant is a property that is preserved under certain kind of transformations such as: thinning, thickening, resizing, rotating, morphing, etc. For this reason, many computer applications in different areas (medical image, cartography, computer graphics, molecular modeling to name but a few [5]) involve the computation of this kind of invariants. In general, the application of topological methods produces a significant reduction in the amount of data, since the emphasis is on shape as opposed to metric.

The number of connected component is one of the most basic topological invariant for digital images. This feature is coded by the *Betti number* β_0. The number of "gaps" that separate components in 2D is coded by β_1. Extending to higher dimension, in 3D, the number of connected components, "tunnels" that pass through the shape and "voids" that are connected components of the complement space inaccessible from the outside, are coded by β_0, β_1 and β_2, respectively.

It is easy to define and count connected components, gaps and voids. The concept of a tunnel and how to count them is somewhat more delicate. For example, consider a set of 6 edges joined to form the skeleton of a tetrahedron. We can think that there are 4 tunnels (the 4 triangles) but β_1 is only 3. The reason is that only 3 of the tunnels are independent. The 4th tunnel is a combination of the independent ones. Connected components, gaps, voids or tunnels are classified in equivalent classes. These equivalent classes form groups called *homology groups*. Fixed q, the classes of the homology group of dimension q are generated by independent *q-cycles*. Roughly speaking, a connected component is a 0-cycle, a tunnel (in 3D) or a gap (in 2D) is a 1-cycle and a void in 3D is a 2-cycle. The rank of the homology group in dimension q is the Betti number β_q. The advantage of using homology instead of simply Betti numbers is that homology characterizes the holes, not only counts them. Think in a hollow torus. Its Betti numbers are: $\beta_0 = 1$, $\beta_1 = 2$ and $\beta_2 = 1$. That is, the torus has one connected components, two tunnels and one void. In order to find out, for example, the tunnels, we compute the representative 1-cycles to obtain one tunnel that pass through the hole of the torus and another one (less intuitive) that surrounds the cavity.

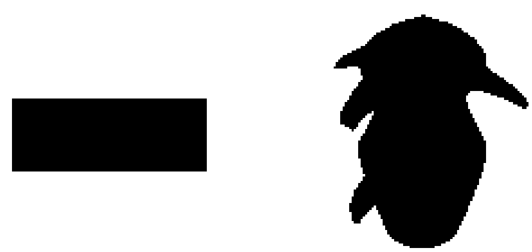
Nevertheless, sometimes it is not enough to compute homology in order to characterize shape. Think for example in the letters A and O. They have both the same homology (in fact, they are homotopic). So, how can we distinguish them?

Dealing with rigid objects, the idea could be to consider the topology of the difference between an object and its convex hull, since this information is independent on rigid transformations of the object.

3 The Topology of Convex Deficiencies

The **convex hull** H of an object O is the minimal convex set containing the object [17] i.e. $\nexists H'$ convex, $O \subseteq H'$ s.t. $H' \subset H$. It is also common to call convex hull the boundary of the minimal convex set containing the given object [2] i.e. ∂H. We consider the first definition, in which the convex hull of an object has the same dimension of the object, and a convex object coincides with its convex hull (See Fig. 1 with digital images from Kimia-99 database[16]).

In this paper the convex hull of the points in R2 defined by the centers of the pixels of the "binary shape" is computed. A pixel is considered *inside* the convex hull if its center lies inside or on the computed convex hull borders. In the case of the boundary graph representation of the image, all points around a foreground pixel are considered to compute the convex hull. In the same way, a point around a pixel is considered inside the computed convex hull if its coordinates lie inside or on the computed convex hull borders.



a) b)

Figure 1: a) Convex shape coincides with its convex hull. b) Non convex shape.

Previous work use the convex hull to obtain a hierarquical representation of concavities [4, 6] and match 2D shapes [3]. The **convex deficiencies** are the connected components of the difference between the object and its convex hull (has the same dimension of the object, see Fig. 2). In the figure, the digit images are showing 0, 2(1 hole + 1 cavity) and 2(2 cavities) convex deficiencies, K image and human shape show 3 and 5 convex deficiencies respectively. The (non-empty) intersection of the two boundaries (curves in 2D, surfaces in 3D), the one of the object and the one of its convex hull, induces a binary map on each of them. A concavity of the object can be associated with a connected component of non-convex boundary points surrounded by convex ones. A more complex convex deficiency is formed around the neck of a person, between the head and the body. It is a torus like 3D object, and the associated connected component of the non-convex boundary points forms a closed loop.

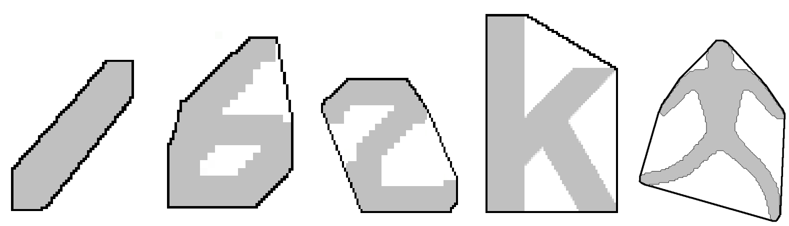


Figure 2: Shapes drawn in gray and its convex hull borders drawn with black lines.

Now we can consider the convex deficiencies, their holes and homology groups. They characterize many useful properties of the object and are independent of the object's pose. Their 'structure' remains the same when the object moves rigidly, even some restricted deformations do not change this convexity-induced topology.

This structure can be related to the projected image: bounded objects map into bounded regions in the image. The convex hull of the 3D object maps into a convex region in 2D which is the convex hull of the projected 2D region. The region's boundary (= silhouette of the object) separates the object's image from its background (except in the presence of occlusions) and can be detected in many cases by segmentation. The silhouette back-projected in 3D also separates the

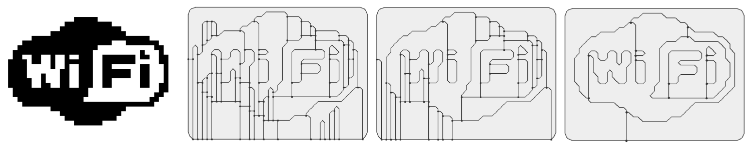


Figure 3: A digital image I, and boundary graphs $\bar{G}_6$, $\bar{G}_{10}$ and $\bar{G}_{16}$ of the pyramid of I.

points that are visible on the 3D object's surface from the points on the back-face of the object. This is another binary mask determined by the object's shape and the viewpoint.

Unfortunately determining the exact geometry of an object is not always possible and depends on the acquisition method. For example if considering a single camera, there are cases where it is not possible do differentiate between an object with two deep concavities and one with a tunnel. Sometimes changing the pose can help to see the connection, but not always, e.g. if the tunnel turns inside the object such that we cannot see its end, as a human we have to use other tricks: sticking something through it, blowing through the "hole", etc.

3.1 Computation of convex deficiencies and their topology

In the following will briefly discuss the computation of the convex hull and of the topology of the convex deficiencies.

Irregular Graph Pyramids: A *region adjacency graph* (RAG), encodes the adjacency of regions in a partition. A vertex is associated to each region, vertices of neighboring regions are connected by an edge. Classical RAGs do not contain any self-loops or parallel edges. An *extended region adjacency graph* (eRAG) is a RAG that contains the so-called *pseudo edges*, which are self-loops and parallel edges used to encode neighborhood relations to a cell completely enclosed by one or more other cells [11]. The *dual* graph of an eRAG G is called a *boundary graph* (BG) and is denoted by $\bar{G}$ (G is said to be the *primal* graph of $\bar{G}$). The edges of $\bar{G}$ represent the boundaries (borders) of the regions encoded by G, and the vertices of $\bar{G}$ represent points where boundary segments meet (See Fig. 3). G and $\bar{G}$ are planar graphs. There is a one-to-one correspondence between the edges of G and the edges of $\bar{G}$, which also induces a one-to-one correspondence between the vertices of G and the 2D cells (will be denoted by *faces*[2]) of $\bar{G}$. The dual of $\bar{G}$ is again G. The following operations are equivalent: edge contraction in G with edge removal in $\bar{G}$, and edge removal in G with edge contraction in $\bar{G}$.

A (dual) irregular graph pyramid [11, 12] is a stack of successively reduced planar graphs $P = \{(G_0, \bar{G}_0), \ldots, (G_n, \bar{G}_n)\}$. Each level $(G_k, \bar{G}_k), 0 < k \leq n$ is obtained by first contracting edges in G_{k-1} (removal in $\bar{G}_{k-1}$), if their end vertices have the same label (regions should be merged), and then removing edges in G_{k-1} (contraction in $\bar{G}_{k-1}$) to simplify the structure. The contracted and removed edges are said to be *contracted* or *removed* (sometimes called *removal* edges) in $(G_{k-1}, \bar{G}_{k-1})$.

The presented dual graph pyramid can be used to correctly encode the topology of a 2D manifold[3]. In the 3D and higher dimensional cases, other structures like combinatorial maps [9] or generalized maps [8] can be used to represent the levels.

Topological Invariants in a Pyramid: In [7] it is shown that topological invariants computed on different levels of a graph pyramid constructed as mentioned before are equivalent and that one can construct the pyramid such that all levels are homeomorphic or homotopic (depending on the desired properties). Representative cycles [15] of the homology groups and representative

[2]Not to be confused with the vertices of the dual of a RAG (sometimes also denoted by the term *faces*).

[3]Using this representation the graphs G and $\bar{G}$ can be no longer *planar* but for each of them there exists an embedding on the 2D manifold such that edges cross only at their endpoints. This apply because a 2D manifold is locally isomorphic to an Euclidean 2D space.

Table 1: Classifying Capital Letters.

class	holes	concavities	letters
1	0	0	I
2	0	1	C, G, J, L, U, V
3	0	2	E, F, H, N, S, T, Z
4	0	3	K, W, M, Y
5	0	4	X
6	1	0	D, O
7	1	1	A, P
8	1	2	Q, R
9	2	1	B

cocycles [7] of the cohomology groups of an object can be efficiently computed using an irregular graph pyramid.

Given the convex hull, with the mentioned algorithms, it is enough to build the pyramid once. This pyramid can then be used to efficiently compute the topological invariants for all connected components of the object and the convex deficiencies.

Convex Hull in a Pyramid: The concept of convex hull is not new, and quite a few computation and approximation algorithms exist, e.g. [2, 10, 13]. A method to compute the convex hull using a pyramidal approach where computation at some level is followed by a set of projections on the lower levels until the base is reached has been proposed [1]. This algorithm is based on the computation of the OR-pyramid (regular pyramid) on the input binary image. Since, the result is an approximation of the convex hull considering n straight lines which are tangent to the edges of the object, they obtain a large number of false concavities. Also, the vertexes obtained are not a subset of the original polygon.

Computing a pyramid representation that encodes the convex hull region could allow to compute the topological description of the convex deficiencies together with the object topology without any other precomputation.

3.2 Classification Example

Table 1 shows the discrimination capabilities of the number of holes together with the connected components of the difference between an object and its convex hull in 2D, related to concavities. The input data consists of the letters of the alphabet. Nine classes can be discriminated, as opposed to three if using only the holes.

4 Conclusions

Convex deficiencies are the connected components of the difference between an object and its convex hull. This paper proposed to use the topology of convex deficiencies in addition to the topology of an object for doing classification and gave a simple example. An open problem is the extension of algorithms to compute topological invariants of the object in irregular graph pyramids [7, 15] to 3D and higher dimensions. There is an open issue on how to evaluate the reliability of a convex deficiency to overcome their sensitivity to small variantions. e.g. by using metric properties of the volume, thickness, eccentricity of the boundary of the convex hull, etc.

An open problem would be to study the dynamics of the CDs under continuous deformations, e.g. under articulation. Its produces changes in its geometry that for several time intervals keeps the same structure. In the case of articulated objects observations like a human elbow or a knee always show the concavity on the same side are interesting features to describe shapes.

Depending on the acquisition method, the form and thus the precise topology of the object might not be known (e.g. a cavity in a 3D object cannot be “seen”). Thus the relations between the visible parts of convex deficiencies (e.g. their existence and number of connected components) and their real topology have to be investigated.

It is known that the topology of an object is sensitive to certain kinds of noise (e.g. Salt & Pepper). How does this noise affect the convex deficiencies. Also what are convex deficiencies stable over a whole class of objects and which are just exceptions/noise.

Another type of commonly sought features are pronounced protrusions and intrusions. A protrusion of a shape is an intrusion of the complement space. Roughly speaking, intrusions or pockets can be defined as regions in the complement with limited accessibility from the outside. We plan to study conditions under which convex deficiencies are related with pockets.

Computing the convex hull of an object using a irregular graph pyramid is still an open problem. Having such an algorithm, could allow efficient computation of the desired topological invariants, directly on the created pyramid.

Another task will be the computation of the cohomology ring of the convex deficiencies of the given object. We think that in 4D this computation will help us to a finner classification than using only homology.

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