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A concept for shape representation with linked local coordinate systems

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Abstract.

This paper discusses a concept for the representation of n-dimensional shapes by means of a model, based on linked local coordinate systems. Through application of the medial axis transform (MAT) and decomposition of the resulting medial axis (MA), articulated, as well as non-rigid abstract n-dimensional bodies can be described by defining corresponding local coordinate systems for each element. This should allow a distinct and invariant representation of every point of the shape, which can be used for complex composite transformations of the object in the context of robotic manipulation.

1. Introduction

For the automatic manipulation of objects and reasoning considering their attributes, a powerful model is needed. Articulated objects, like the human body, or deformable objects, like a piece of clothing, demand a model that is able to represent complex intrinsic transformations. These classes of objects can be represented by defining coordinate systems for each segment, so every point of the object is distinctly determined by a set of coordinates. One application, for both classes of objects mentioned, is automated dressing-assistance for a person. Linked local coordinate systems should allow the description of every point of the shape, so it can be exactly defined where a robotic arm needs to grasp a glove and how it needs to place it for the person to slip in comfortably, considering the person's range of motion.

A coordinate system is specified by its origin, determining the location, and a set of basis vectors, defining the orientation and scale of the element. It makes the description of an element invariant to changes. In the case of articulated movements, the specific coordinates of the parts do not need to be changed. The intrinsic movement of an articulated object's element can be described as a transformation between two linked coordinate systems. Swinging of the arm can be characterised as a transformation of the arms coordinate system in respect to the linked coordinate system of the torso for movement of the shoulder and transformation of the distal part's coordinate system in respect to the system of the upper arm for movement of the elbow. The coordinate system of the hand in respect to the system of the forearm does not change in that case (Fig. 1). In case of a smooth deformation, local interpolation between the transition of the elements may be needed.

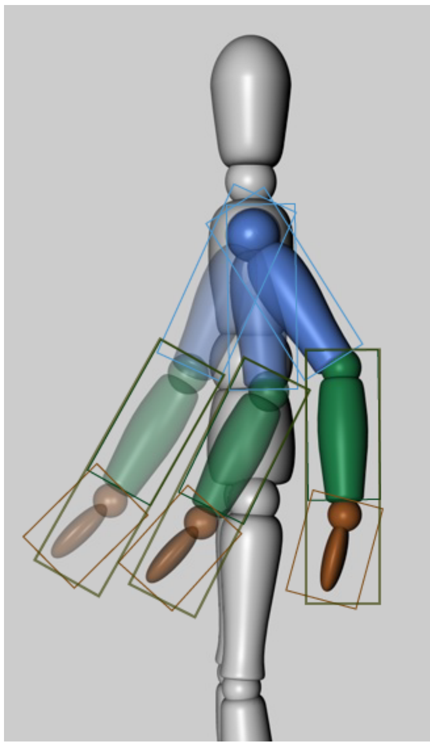

Figure 1. Linked local coordinate systems of a swinging arm. Frames indicating the area of a coordinate system. Forearm and hand do not move in respect to each other while the linked system of the distal part (parent of hand and forearm) of the arm changes in respect to the system of the upper arm.

The intrinsic movement of a non-rigid object is supported by the model's invariance to deformation originating from the axial representation. The object's axial representation provides the linked local coordinate systems. In 3D space, axial representations can be produced by sweeping spheres along the axis [16]. For 2D objects, geometric primitives, like circles or line segments, can be used as generators [14, 20]. The linked local coordinate systems are based on the resulting medial axis of the object using an end point as the origin and a branch of the medial axis as a basis vector of the coordinate system.

Several problems need to be addressed to provide a stable and invariant model that can represent an object and leads to reliable reasoning:

1. Noise
   - Noise inside the shape creating holes.
   - Noise along the boundary creating spurious branches.
2. Decomposition
   - Multiple affiliation of points in branching areas.
3. Preservation of structure
   - Ordering of axes at branching points.
4. Special shapes
   - Spheres and objects based on spheres.
   - Circular MAs.

The novelty of the method is the utilisation of linked local coordinate systems for the representation of n-dimensional objects for robotic manipulation.

The paper is organised as follows. In section 2, related work is outlined. Section 3 describes the proposed method and its open problems in detail. Section 4 concludes the paper with a discussion of the method.

2. Related Work

Most recently, research in the field of robotic dressing-assistance was done by Gao, Chang and Demiris, who utilise randomized forests for a model of the upper body [6]. Klee et al. used a skeleton tracker for a robotic dressing-application.

Handling and predicting articulated objects or non-rigid objects demands a complex model that can represent the vast amount of different possible appearances of an object. Several projects have already been dedicated to that issue. Li, Chen and Allen [11] used meshes of deformable objects to simulate the movement and its results to identify grasping points of garments. With a system of dictionary learning via spatial pyramid matching and sparse coding, a robotic grasper is enabled to grasp, flatten and fold garments. Felzenszwalb, Mc Allester and Ramanan [5] published an algorithm for the recognition of deformable objects in images by means of a discriminatively trained, multiscale, deformable part model in 2008. Godec, Roth and Bischof [7] described hough-based tracking of non-rigid objects in 2013. Their approach utilises the generalised Hough-transform to handle articulated and non-rigid objects. Pouch et al. [13] resort to the MAT to segment the deformable aortic valve apparatus in 3D echocardiographic images.

To provide a stable basis for the concept, a MAT algorithm must be used which can provide a geometrically accurate and compact MA. In recent years, several groups have been dedicated to improve prior efforts in that field. Li et al. published an approach for MAT by Quadratic Error Minimization to compute a stable and compact MA [10] The groups of Zhu et al. published a paper on the constructive generation of the medial axis for solid models [18] and also an approach for calculation of the medial axis of a CAD model by parallel computation [19]. Aichholzer, Aigner, Aurenhammer and Juettler showed a technique for the MAT by means of a polyhedral unit ball instead of the standard Euclidean unit ball [2]

3. Method

MAT has the property of producing a MA of one dimension less than the object in many cases. A 3D object creates a 2D MA and a MAT of a 2D MA generates a 1D manifold (Fig. 2.a) that can be decomposed at its branching points (Fig. 2.b, Fig. 2.c). As branching points we denote locations where more than 2 branches of the MA meet. These points represent the basis of convexities of the shape. Points within the largest inscribing circle around these branching points, the branching area, have an unclear affiliation to a MA branch, which poses a problem when the connected MA branches move in respect to each other. The decomposed

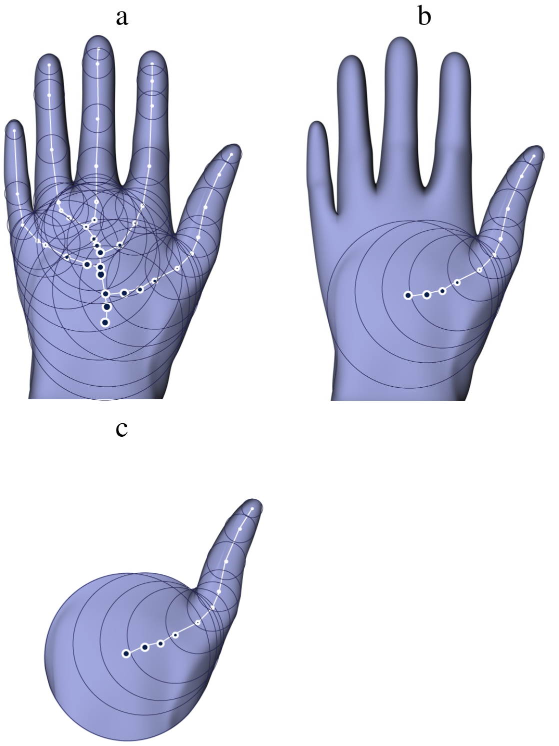

Figure. 2. a) MAT of the image of a hand. Largest inscribing circles form the MA. b) Decomposed branch of the MA. c) Area to be described in respect to this branch.

branch of the MA is straightened to form the x-axis of a new coordinate system by replacing the geodesic distances by Euclidean coordinates. The distances along the MA stay identical, while the curvature is removed (Fig. 3).

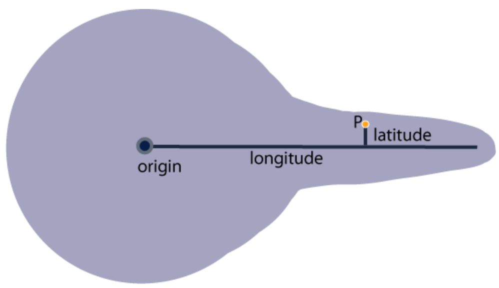

Figure 4. Coordinate system based on a MA branch. A point is defined by longitude and latitude.

This makes the representation invariant to deformation of the object, except stretching and compression, where the geodesic distance may change with movement. One end point of the axis can be chosen as the origin. All points within the silhouette of the object can be described as a tuple of longitude along the axis and latitude as the distance of the point along the normal to the axis (Fig. 4). This procedure

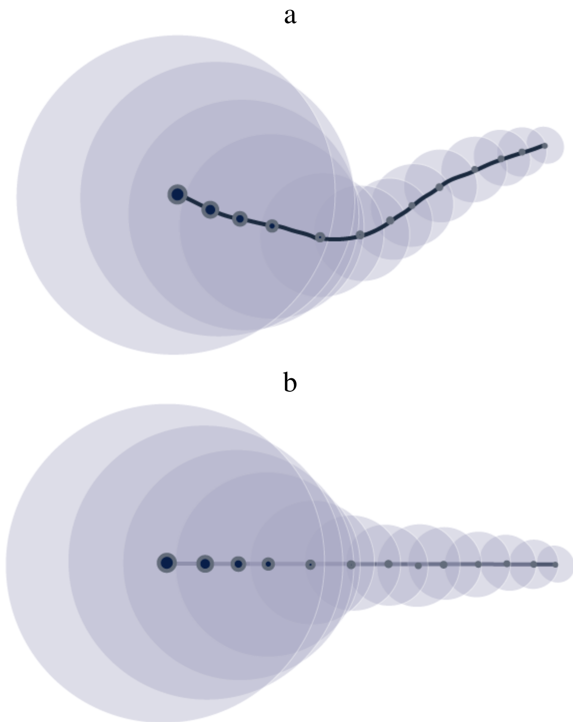

Figure. 3 a) Elongated shape with MA and its straightened representation (b). The representation is invariant to deformation.

of MAT, decomposition and straightening creates a graph with end points and branching points as nodes and axis branches as edges (Fig. 5).

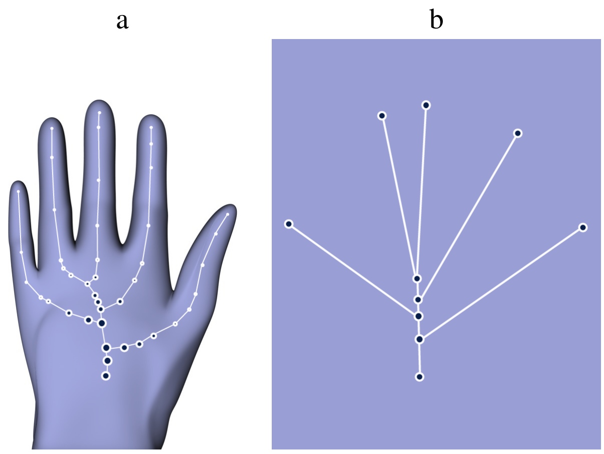

Figure. 5 a) MAT of the image of a hand. b) Graph created by straightening the MA branches of the hand.

By means of the graph, the structure of the object can be identified. The graph concept is based on the notion of cellular complexes, described by Kovalevsky [9], which states that an n-dimensional object is confined by an (n-1)-dimensional object. The

1D MA is confined by 0D points, the 2D MA is confined by 1D curves and so forth. Based on this principle of cellular complexes and the attribute of MAT to produce a MA of the objects dimensionality minus 1 in many cases, it is assumed that the proposed method holds for many n-dimensional objects by recursive application until 1-dimensionality is reached.

To communicate the principle of MA, we show how to build an abstract object from its MA. A shape can be created by sweeping a circle along a 1D Axis as can be seen in Fig. 6. The MA is synonymous

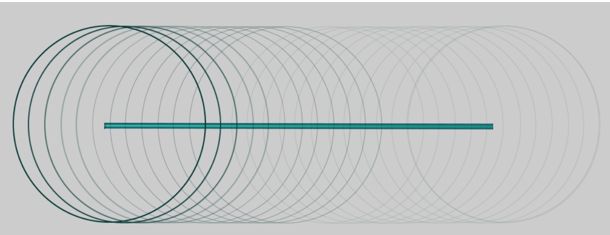

Figure 6. Circles swept along a 1D MA. Transparency indicates the sweeping movement.

with the x axis of a coordinate system we use to define all points of the shape. The radius of every circle at position x along the axis has to be stored to create the intended object. This assures the preservation of shape. Given that the circles have to touch the outline of the shape at at least 2 points at all times and no circle is completely contained in another, the silhouette of all the circles combined describes the shape that is to be produced [3]. Noise on the boundary of the object can cause spurious branches, meaning branches of the MA that do not hold valuable information about the appearance of the shape. Noise within the object may cause holes and therefore circular MAs. In Fig.7, we compose several branches to

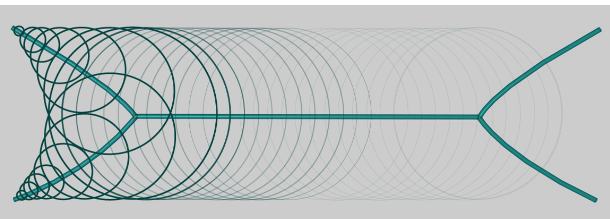

Figure 7. Circles swept along a composed 1D MA. Transparency indicates the sweeping movement.

one MA. The constellation of branches determines the structure of the object. The structure can have different constraints in its movement, depending on the intrinsic mobility of the object. This topic is discussed further in chapter 3.3 Preservation of structure. When creating the 2D object, the Euclidean coordinates of the MA are replaced by the geodesic coordinates the axis shall have within the shape. Fig. 8 shows the 2D object that emerges from the composed MA. This 2D object itself can be used as the MA for a 3D object. This concept can be be continued due to the MAT's attribute to create an object with the dimensionality of the object minus 1. So its reversal leads to an object with the dimensionality of the MA plus 1.

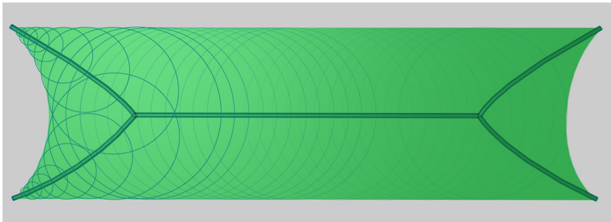

Figure 8. Covered area as 2D MA of a 3D object.

3.1. Noise

Noise on the boundary of the shape can cause spurious branches. Noise within the shape may cause holes, which can lead to circular MAs. Several projects are dedicated to the reduction of the influence of noise on the MAT. Most recently Spitzner and Gonzalez [17] published a method called Shape Peeling to improve the stability of image skeletons. Abiva and Larsson [1] proposed a method to utilise the Scale Axis Transform to prune the MA of spurious branches. Montero and Lang [12] published an algorithm for skeleton pruning by means of contour approximation and the integer MAT in 2012.

3.2. Decomposition

Decomposition is performed in branching areas to obtain less complex axes. Serino, Arcelli and Sanniti di Baja [15] recently described the decomposition of 3D objects at branching points to obtain meaningful object parts. In 2D, the branching area lies within the largest inscribing circle where 2 or more branches of the axis meet in the centre (Fig. 9). While the points of the shape lying in a circle that only belongs to one axis, are uniquely defined, points within the branching area can be described in relation to several branches of the axis (Fig. 10). If branches move in respect to each other, these points shall each be affiliated with only one branch to preserve a unique representation. While Serino, Arcelli and Sanniti di Baja [15] can already demonstrate impressive experimental results of the decomposition of the composed

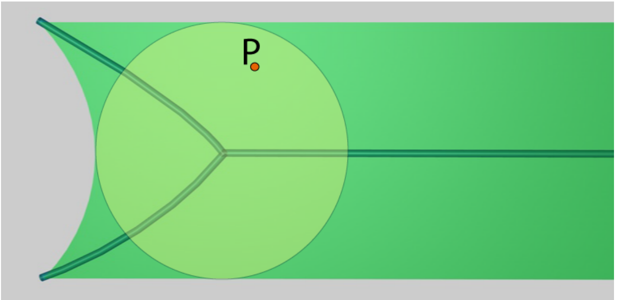

Figure 9. A point within a branching area can be described in relation to several branches of the axis.

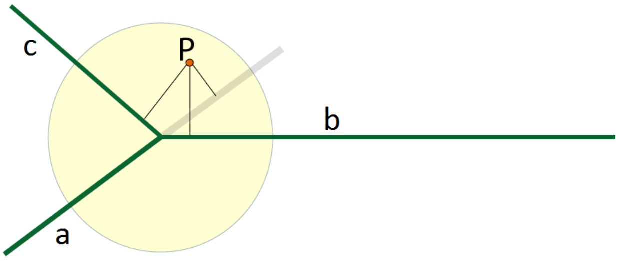

Figure 10. A point within a branching area can be described in relation to several branches of the axis. Axis a is extended across the centre, illustrating its negative domain.

1D MA of 3D objects, MAs of higher dimensions require further research.

In 3D, there can be branching points or branching curves where the branches of the MA meet as can be seen in Fig. 11. In a first idea we approach the branching area as if it is an object itself. The branching area of a curve we define by the largest inscribing sphere that is swept along the branching curve (Fig. 12).

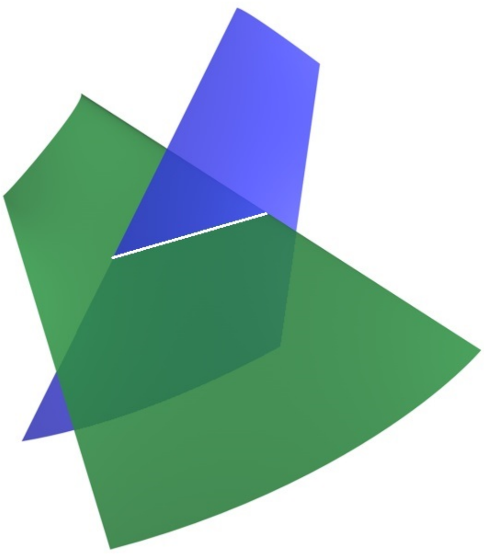

Figure 11. Two 2D MA branches of a 3D object forming a branching curve where they intersect.

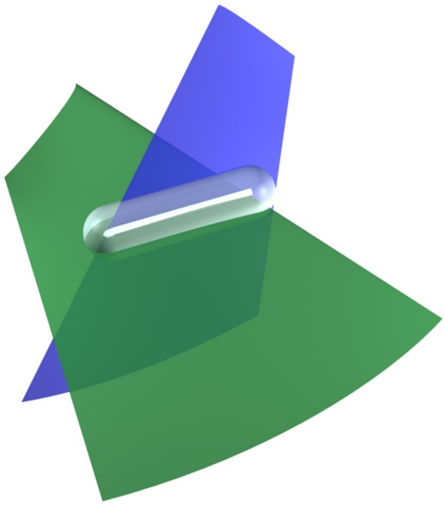

Figure 12. A sphere swept along the branching curve creating a new 3D object based on a sphere.

The branching area itself can be seen as a 3D rod-like object or as a 4D object created by sweeping a 3D sphere along an axis. This implies a leap of at least 2 dimensions to reach the 1D MA, which violates the assumption that the MAT reduces the dimensionality of an object by 1. A problem that is yet to be solved and is explained further in the chapter 3.4.1 Spheres and objects based on spheres.

A different approach is to apply the MAT recursively to every branch of the MA until 1D is reached. This way, joints will not necessarily imply a connection of the MA branches (Fig. 13) and the MA branches of an object might not intersect. If the MA breaks into several pieces, it arises the question of how the structure can be maintained. Further work on this matter is required.

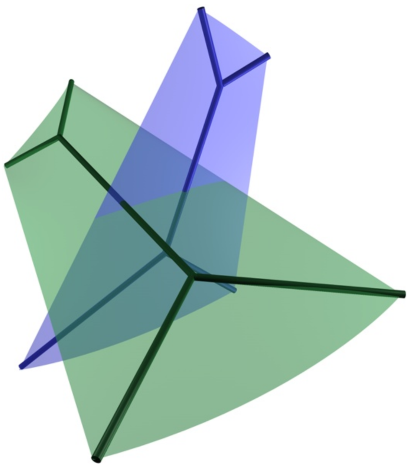

Figure 13. 1D MA branches of the 2D MA branches do not intersect.

3.3. Preservation of structure

Articulated objects with a specific range of motion require constraints at joints, so the human forearm can not rotate around the elbow, but can only flex in one direction to a certain degree. Non-rigid objects, like cloth, require different constraints since they do not have joints, but feature a certain thickness, stiffness, weight and other properties. A basic ordering has to be maintained regardless of these characteristics. As shown in Fig. 14, all MA branches might be able to move freely, provided they do not cross planes spanned by two different axes to sustain the objects organisation. The structure can be preserved by considering the branches of the MA as edges and the end points and branching points as nodes of a combinatorial map as described by Damiand and Lienhardt [4].

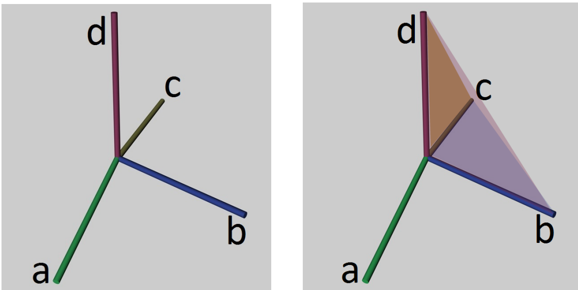

Figure 14. 3D branching point of MA branches. Branch a can move freely except across the triangles spanned by the other branches b, c and d.

3.4. Special shapes

There are several open problems regarding special shapes in the method that require further research. Thoughts of the community on the matter are highly appreciated.

3.4.1 Spheres and objects based on spheres

The concept of MAT is mostly built on the usage of circles and spheres. If an object, or a part of it, itself is one of these primitives or based on the primitive in a higher dimension, the MAT will not create an object of its dimension minus 1, but it may create a MA with a dimensionality even lower. This violates our basic assumption that this is the case. This means that the MA can not be used to determine the location of points of the shape uniquely. One approach to solve this problem is to utilise spherical coordinate systems. Fig. 15.a shows a 3D sphere that creates a 0D MA. Fig. 15.b illustrates the sphere after application of an equator to orient the spherical coordinate system. With these systems, all points of a sphere can be distinctly determined. Objects based on spheres imply that the shape can be created by moving a sphere along a path (Fig. 15.c, Fig. 15.d). It follows, therefore, that every point of the object based on a sphere can be uniquely determined when the spherical coordinate system is moved along the MA.

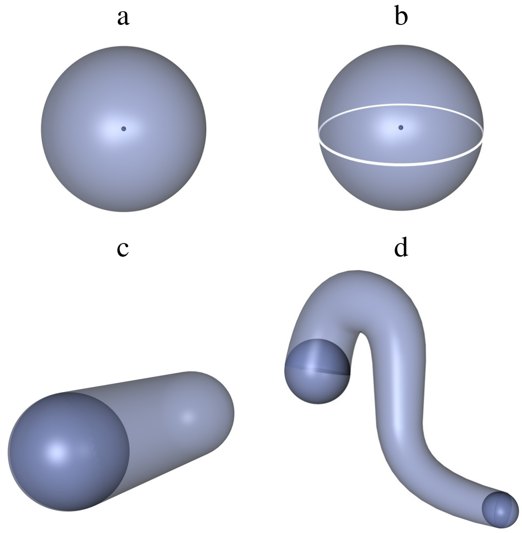

Figure 15. a) 3D sphere producing a 0D MA. b) Equator applied to a sphere to provide orientation for the spherical coordinate system. c,d) Shape described by sweeping a spherical coordinate system along a path.

3.4.2 Circular medial axes

Circular medial axes occur when an object element has genus higher than 0 (Fig. 16.a) and at concavities of the object (Fig. 16.b). If a circular MA branch is connected to 1 or more other branches of the MA, the branching points can be used to decompose the circular MA and therefore create non-circular sections that can be treated regularly. This is the case if the object features a tail. Elements with genus higher than 1 also feature connected MA branches because of the bridge between the holes whose MA branch connects the sides. This leaves an issue for objects with genus 1 and no tail (Fig. 16.a) and objects with convex elements (Fig. 16.b). The n-dimensional MA is not confined by a (n-1)-dimensional object, which violates one of the basic assumptions of this method, namely the concept of cellular complexes. If an object produces a circular MA without connected

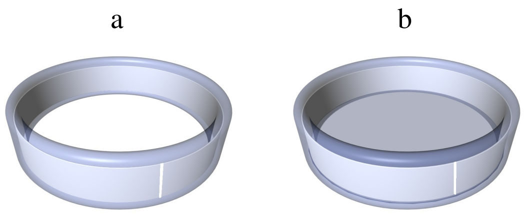

Figure 16. a) 2D circular MA within a tube-like object with an arbitrarily set reference (white). b) 2D circular MA branch as part of an object's MA with an arbitrarily set reference (white).

branches, there is no reference point that can be used as the origin of the coordinate system. A first attempt to solve this problem, based on the findings of Illetschko [8], is to place an arbitrary reference point. This point can be used as the origin of the coordinate system based on the MA. Depending on the dimensionality of the object, also a cut can be necessary. Points within the area of the new origin can then be defined in relation to both end points of the MA.

A special case is shown in Fig. 17. The torus is a shape based on a sphere, meaning that it can be described as a sphere moved along a circular path. As explained earlier, this enforces the use of a spherical coordinate system. Also the torus has a circular MA, which requires an arbitrarily set reference point.

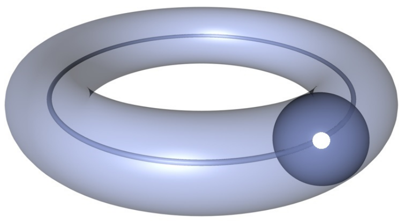

Figure 17. Special case: Torus is a shape based on a sphere and creates a circular MA. From arbitrarily set reference point on the MA (white), a spherical coordinate system is swept along the MA.

4. Conclusion

This paper proposes an novel concept for the representation of n-dimensional shapes through a model, based on linked local coordinate-systems. Through recursive application of the MAT and decomposition of the resulting MA, some n-dimensional objects can be reduced to multiple 1-dimensional sub-elements that are used as the axis for coordinate-systems. The 1D elements as edges and their end points as nodes, form a graph that represents the object. Articulated, as well as non-rigid objects can be described by defining corresponding coordinate systems of each element. This should allow complex composite transformations of the object. Intrinsic movement does not imply the transformation of point-clouds or meshes, but of linked local coordinate systems.

Further work to be done on the project is to provide a proof of concept, especially concerning the feasibility of the method for n-dimensions and resolution of the open problems described in this paper.

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