Estimating Graph Topology from Sparse Graph Signals with an Application to Image Denoising

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Abstract—**Graph signal processing is a framework that allows us to work with general unstructured discrete data that cannot be handled with classical Discrete signal processing. The underlying graph topology plays a crucial role in determining the definition of Fourier transform on graphs. Graph topology for a given graph signal is not always available and may also not be unique. In this paper we address the problem of estimating graph topology from signals that are sparse in the frequency domain. We estimate the graph Laplacian matrix in an optimization framework that minimizes errors in relations known to exist between the graph signals and their Fourier transforms. We also propose to use this algorithm for adapting an existing graph based non-local image denoising algorithm, which is known to perform well only for piece-wise smooth images. We provide results on natural, texture and smooth images that support our claim that with our topology estimation algorithm the denoising algorithm is able to adapt to different image structures. We compare our results with the graph based non-local method and the state-of-art BM3D algorithm, using different performance measures.**

I. Introduction

With the development of Graph signal processing over the last few years, graph-based modeling of various forms of discrete signals[1], from uniformly sampled 1D signals to social network data has become popular[1], [2]. The underlying domain in some cases is unstructured and may have complex interrelations. Graphs, defined as a collection of vertices and edges, have provided an appropriate modeling choice in these scenarios. In such cases, data can be represented as collection of samples with one sample at each vertex, and are typically called graph signals. The weight of the edge connecting two vertices contains information of their intrinsic relation. For instance, the edge weight may be inversely proportional to the geographical distance in case of climatic data or it may be directly proportional to similarity of intensity for neighboring pixels in an image.

There have been two popular approaches for Graph signal processing, one based on graph Laplacian (spectral graph theory)[1], the other using the adjacency matrix (Algebraic signal processing) [2]. Both approaches generalize a few properties of the Discrete Fourier Transform, and have their own set of advantages and disadvantages. In both cases, the graph topology plays a crucial role in the definition of the Graph Fourier Transform (GFT).

For many graph signals, the graph topology is not known, and an assumption based on the source of the data is made about the topology. For instance, in case of temperature at weather stations, stations within some geographic distance are connected with an edge with weight being inversely proportional to the geographic distance. While in case of digital images, pixels in the 8 neighborhood are connected with an edge with weight proportional to the similarity of the color. Thus, there are several choices of graph topology available, and a right choice needs to be made depending on the application.

A data-driven approach is to choose(optimize) adjacency relations between vertices such that it satisfies assumptions made on the graph signals, for example, the signals being low frequency (smooth). Following this strategy, in this paper, we focus on learning the graph topology from a collection of graph signals which are assumed to be sparse in the frequency domain. It is well known that classes of signals like audio and images are sparse is frequency domain, hence the assumption is practically motivated. The estimation of the underlying graph topology is modeled as a matrix estimation problem, and we attempt to solve it using an optimization framework.

We also discuss an application of our topology estimation algorithm for image denoising. Hu et al. [3] proposed a *Non-local Graph based Transform* approach (henceforth denoted by NLGBT) for depth-image denoising. In this patch based method, the edge-weights between pixels for any patch was assumed to be proportional to the similarity of the color as discussed earlier. This was able to model the piece wise smooth structure in a depth image, but is unable to adapt to more diverse structures found in natural and texture images. We use our topology estimating algorithm to adapt their denoising algorithm for more diverse images.

[1]Technically, graphs represent the domain of the discrete signals.

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The paper is organized as follows: We first discuss related work in estimation of graph topology, in the next section. In section 3, we describe the proposed algorithm for learning graph topology under the assumption that the given collection of graph signals have a sparse frequency domain representation. Section 4 provides details of our adaptive Non-local Graph based image denoising algorithm. In section 5, we describe our experiments for estimating graph Laplacian, and image denoising. We end this paper with a conclusion and discussion along with possible extensions.

II. Related Work

A graph $\mathcal{G}$ is a collection of N vertices (sites) $S = \{1, \ldots, N\}$ with edges between vertices represented in an *adjacency matrix* $A \in \mathbb{R}^{N \times N}$ such that each entry A_{ij} contains the edge-weight of the edge between vertices i and j. $A_{ij} = 0$ indicates the absence of the corresponding edge. For undirected graphs, A is symmetric, the diagonal matrix D denotes the *degree matrix*: $D_{ii} = \sum_{j=1}^{N} A_{ij}$, and $L = D - A$ denotes the *unnormalized graph Laplacian* (henceforth Laplacian). A real-valued graph signal is a function $x : S \rightarrow \mathbb{R}^N$, such that x_i represents the signal value at vertex i. A collection of m real-valued graph signals is given by a matrix $X \in \mathbb{R}^{N \times m}$, where each row i corresponds to the slice of the collection on vertex i, while each column k corresponds to the k^{th} signal on the graph. The k^{th} graph signal from the collection will be denoted by $X[k]$, while the k^{th} graph signal at vertex i will be denoted by $x_i[k]$. For an undirected graph $\mathcal{G}$, the Laplacian L is always diagonalizable with the matrix of eigenvectors $V = [v_1 \ldots v_N]$: $L = V \Lambda V^T$, where Λ is a diagonal matrix of eigenvalues. The matrix V^T is defined as the GFT matrix. For directed graphs, GFT is defined as the matrix V^{-1}, where V is the matrix of generalized eigenvectors V that transforms the adjacency matrix into its JNF J: $A = VJV^{-1}$, as shown in [2]. It is thus clear that graph topology plays a crucial role in determining GFT and therefore other signal processing concepts, tools and algorithms.

Surprisingly, estimating the graph topology has not received the attention it deserves. In case of graph signals from geographical sensors, typically sensors within a particular distance of each other are connected with an edge which has an edge weight inversely proportional to the geographical distance, or for blogs on the internet, they are connected based on hyperlink references to each other[2]. In case of patches in an image, each pixel is modeled as a vertex of a graph and neighboring pixels are connected with an edge-weight proportional to similarity of the color at the vertices under consideration[3]. In [4], the authors model the graph signals as samples drawn from a Gaussian MRF, in which case they assume the adjacency matrix of the graph to be the precision matrix (inverse of the covariance matrix) estimated from the collection of graph signals.

In [5], graph signals are assumed to be generated via a causal time series process. The k^{th} graph signal is assumed to be a result of a linear recurrence relation between previous q graph signals and noise: $X[k] = \sum_{i=1}^{q} P_i(A, c) X[k-i] + W[k]$, where $P_i(A, c)$ is a graph filter expressed as a polynomial in the adjacency matrix with coefficient vector c, while $W[i]$ is a random noise process. The algorithm is based on an optimization framework that minimizes the error between the two sides of the above model.

Assuming that the given collection of graph signals are low frequency(smooth), Dong et al.[6], propose a method to estimate the Laplacian matrix. Intuitively, a smooth graph signal has similar values on strongly connected vertices. Thus, for a given graph signal x, lower the value of $x^T L x = \frac{1}{2} \sum_{i \sim j} A_{ij} (x_i - x_j)^2$, smoother will be the graph signal. By posing the above constrains in an optimization problem that encourages sparser Laplacian estimates, a graph Laplacian that describes the given graph signal collection using lower frequency basis is estimated. In [7], the authors propose an approach to recover adjacency and Laplacian matrix from the spectral templates. The authors assume that the eigenvectors of the ground truth Laplacian is available, which simplifies the problem to a great deal. Estimation of graph topology (adjacency or Laplacian matrix) can be expressed as an estimation of kernel $K \in \mathbb{R}^{N \times N}$ which maps the relation between the graph signals at two different nodes on $\mathbb{R}$. Different assumptions on the graph signal imply a different kernel and provides a different topology as explained in Chapter 8 of [8].

For many signals, for example images, the high frequency components are present and convey important information. Hence the assumption of smoothness is violated in such cases. It is more appropriate to assume that the signal is sparse in an appropriate basis. In this work, we try to estimate the graph topology, given a collection of signals X which are assumed to be sparse in the frequency domain (GFT domain). We show results on simulated data, and as an application we show that using the topology learnt from our algorithm, one can adapt the NLGBT image denoising algorithm to different classes of images.

III. Proposed Work

Let $X \in \mathbb{R}^{N \times m}$ be the given collection of m graph signals on an N vertex graph. Each of the m signal is assumed to be sparse[2] in the frequency domain with no more than k frequencies contributing. Let $H = V^T X \in \mathbb{R}^{N \times m}$ denote the matrix of GFT coefficients of each signal in X. The unknows in our case are H, V, Λ and

[2]Typically, we consider $30 - 50\%$ nonzero elements.

L. The relation between the unknowns and the known - the collection X are:

$$X = VH \tag{1}$$

$$X^T L X = H^T \Lambda H \tag{2}$$

These two relations yield the following optimization problems to estimate the unknowns:

Problem 1: Estimating GFT and sparse GFT coefficients.

$$(\widehat{V}, \widehat{H}) = \arg \min_{V \in O(N), H} ||X - VH||_F^2 + \beta ||H||_1, \tag{3}$$

where $O(N)$ denotes the set of all $N \times N$ real orthogonal matrices, $|| \cdot ||_F$ and $|| \cdot ||_1$ denotes the Frobenius norm and the ℓ_1 norm of the argument (column-wise ℓ_1 norm if the argument is a matrix), respectively.

Problem 2: Estimating Laplacian.

$$\widehat{L} = \operatorname*{argmin}_L ||H^T \Lambda H - X^T L X||_F^2 + \gamma ||L||_F^2 \tag{4}$$

$$s.t. \quad L_{ij} = L_{ji} \leq 0, \quad \forall i \neq j,$$

$$trace(L) = N, \quad L.1 = 0,$$

where γ is the parameter for controlling the sparsity. The constraints on L ensure that it has the properties of a typical graph Laplacian: symmetric, positive definite, and are borrowed from [6]. A sparser Laplacian estimate is preferred by adding a sparsity cost and the condition on trace eliminates trivial solutions. The eigenvalue matrix Λ used in Equation (4) is the eigenvalue matrix of an intuitively chosen initial Laplacian matrix L_0.

We now discuss the details of the two problems given above.

A. Estimation of Eigenvector basis and frequency domain representation H

The eigenvector matrix V and the GFT coefficient matrix H are obtained by solving Problem 1 with an alternating minimization strategy, i.e, fixing one variable estimating the other, and vice versa. Assuming an estimate $\widehat{H}$ for the GFT coefficient matrix H, the eigenvector matrix V can be estimated by solving:

$$\widehat{V} = \arg \min_{V \in O(N)} ||X - V\widehat{H}||_F^2.$$

The solution to this problem is well-known and is obtained via the SVD of the matrix $\widehat{H}X^T$. Given that P and Q be the left and right singular vector matrices of $\widehat{H}X^T$, the estimate of V is $\widehat{V} = QP^T$, as shown in [9]. Equipped with an estimate of V, we solve the following sparse optimization problem to estimate H:

$$\widehat{H} = \arg \min_{H \in \mathbb{R}^{N \times m}} ||X - \widehat{V}H||_F^2 + \beta ||H||_1 \tag{5}$$

Note that the sparsity regularizer is applied to each column of H (GFT coefficient vector of each signal in the collection), hence each column of H is optimized independently. This ℓ_1 - regularized least-square optimization is solved using Gradient Projection for Sparse reconstruction (GPSR) [10]. Methods like Matching pursuit (MP), orthogonal matching pursuit (OMP) [11] or block coordinate descent method can also be used for the same. Eigenvector matrix $\widehat{V}$ and the frequency domain representation $\widehat{H}$ are alternatively minimized till the error $||X - \widehat{X}||_2$ converges, where $\widehat{X} = \widehat{V}\widehat{H}$.

B. Estimation of the Laplacian matrix L

Using the relation $x^T L x = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} (x_i - x_j)^2 A_{ij}$, and comparing only the diagonal elements of $X^T L X$ with diagonal elements of $H^T \Lambda H$, we rewrite the optimization problem in Equation 4 as follows:

$$\widehat{vech(A)} = \arg \min_{vech(A)} ||diag(H^T \Lambda H) - B \; vech(A)||_2^2 + \gamma ||vech(A)||_1, \tag{6}$$

where $vech(A)$ is the column concatenation of the lower triangular subset of the adjacency matrix, and $B \in \mathbb{R}^{m \times \frac{N(N-1)}{2}}$ is the following matrix:

$$B = \begin{bmatrix} d_{12}^1 & \dots & d_{1N}^1 & d_{23}^1 & \dots & d_{2N}^1 & \dots & d_{(N-1)N}^1 \\ d_{12}^2 & \dots & d_{1N}^2 & d_{23}^2 & \dots & d_{2N}^2 & \dots & d_{(N-1)N}^2 \\ \vdots & & \vdots & \vdots & & \vdots & & \vdots \\ d_{12}^m & \dots & d_{1N}^m & d_{23}^m & \dots & d_{2N}^m & \dots & d_{(N-1)N}^m \end{bmatrix},$$

where $d_{ij}^k = (x_i[k] - x_j[k])^2$. This reduces the constraints of symmetry and then Equation 6 can be solved using GPSR [10]. The adjacency matrix A and the graph Laplacian matrix can be later recovered using the duplication matrix M_{dup} as mentioned in [12] :

$$M_{dup}.vech(A) = vec(A) \quad \text{and} \quad L = D - A,$$

where M_{dup} is a matrix which produces $vec(A)$ - the vectorized form of symmetric adjacency matrix A from the lower triangular matrix $vech(A)$. The complete algorithm is summarized as Algorithm1 Before discussing the experiments, we discuss an application of Algorithm 1 for adaptive image denoising.

IV. Application in Image denoising

Image denoising is one of the classical problems in image processing. Recent advances in this field are based on non-local patch based approaches [13]. In this paradigm, denoising is performed by first collecting similar patches throughout the image, transforming the ensemble or the average via a suitable transform and using various thresholding algorithms in the transformed domain.

A. Nonlocal Graph based Image Denoising

Authors in [3] propose a graph based image denoising method, which we summarize for sake of completeness. To begin with, for every (overlapping) patch in the image, similar patches are clustered from which

Algorithm 1 Estimating Laplacian from sparse frequency domain signals

1: **Input:** Graph signal X and parameters β, γ.
2: **Initialize:** $i \leftarrow 1$. L_0 as a natural graph Laplacian, find its eigenvector matrix V and eigenvalue matrix Λ using eigenvalue decomposition and calculate $\widehat{H}_0 = V^T X$
3: **while** $||X - \widehat{X}||_F^2$ decreases **do**
4: $\widehat{V}_i = QP^T$, where Q and P are obtained by SVD of $H_{i-1}X^T$
5: Solve for $\widehat{H}_i$ using equation (5)
6: $\widehat{X} = \widehat{V}\ \widehat{H}$
7: **end while**
8: Estimate Laplacian $\widehat{L}$ by solving equation (6)
9: **Output:** Laplacian matrix $\widehat{L}$ for graph signal X

an average patch is computed. A graph is constructed for the average patch by connecting the 8 neighbors of every vertex (pixel) with an edge having weight proportional (Gaussian kernel) to the similarity of colors of the corresponding pixels. Using the GFT via the Laplacian of this graph, GFT coefficients are computed for every patch in the collection. Hard thresholding via spectrum shrinkage [14] on these coefficients leads to denoising. The denoised patches are then reconstructed by applying the inverse GFT. Since each pixel belongs to several denoised patches, the final pixel is computed using a weighted averaging of all computed denoised patches with weight being proportional to the sparsity of the thresholded GFT coefficient. Filtered noise is then added back to the denoised image to retain the edge information as given in [15]. For details, see [3].

B. Proposed approach: Adaptive NLGBT

The approach mentioned by authors in [3] works well for piece wise smooth images, for example depth images, but fails to perform well with natural and texture images. The main reason for this is that the chosen graph topology fails to adapt to the underlying image structure. We propose to use the graph learnt for each cluster of similar patches using Algorithm1. The transform used for denoising is then given by the GFT defined using the Laplacian provided by Algorithm1. The motivation behind learning the Laplacian matrix is to adapt the underlying image structure for the graph based image denoising algorithm. In order to reduce the number of variables, we restrict Algorithm1 to learn edge weights only for 8 neighbors for each pixel. The value of β to be used in Algorithm1 is empirically set to be inversely proportional to the number of edges in the NLGBT method with edge weight below a threshold obtained by Otsu's method. Intuitively, edges with lower weights indicate presence of discontinuities and hence higher number of contributing frequencies. The adaptive NLGBT algorithm (henceforth ANLGBT) for image denoising is given in Algorithm2.

Algorithm 2 Adaptive NLGBT (ANLGBT)

1: **Input:** Noisy image y, scalar δ for Iterative regularization, number of iterations *iter*.
2: **Initialization:** $\hat{y}^{(1)} = y$;
3: **for** $k = 1$ to *iter* **do**
4: Step A. Patch Clustering
5: Step B. Estimate number of contributing frequencies for setting sparsity of H
6: Step C. Learn Laplacian matrix for the cluster considering 8 adjacent neighbors using algorithm 1
7: Step D. Transform spectrum shrinkage
8: Step E. Image update
9: Step F. Iterative regularization:
$$\widehat{y}^{k+1} = \widehat{y}^k + \delta(y - \widehat{y}^k)$$
10: **end for**
11: **Output:** The denoised image

V. Experiments

We first describe the experiments for estimating graph topology for graph signals that are sparse in the frequency domain, followed by results obtained using the ANLGBT algorithm.

A. Estimating Graph topology

Our experiments are performed on randomly generated graphs with 10 vertices ($N = 10$), with 35% randomly selected edges from the complete graph present. The edge weights are generated using a uniformly distributed random number. The graph signal collection itself is generated by randomly generating sparse columns in H, with sparsity varying between $30 - 50\%$, from which the signal collection is obtained using $X = VH$, where V is the eigenvector matrix (inverse GFT) of the ground truth graph Laplacian. The initial Laplacian is the precision matrix computed from the graph signals. Two examples of ground truth graphs and graphs estimated using Algorithm 1 with $m = 1000$ are shown in Figure1, from which it is evident that the algorithm does a good job in determining the graph topology. We next evaluate the performance of our algorithm for different values of m, i.e., the number of graph signals in the collection. We use four different performance measures for this purpose: precision, recall, f-measure and relative Frobenius error in L. One can expect more accuracy with more data. Average measures obtained over 100 experiments for $m = 500, 1000$ and 1500 each, are tabulated in Table I. It would be incorrect to conclude that our algorithm yields a high error in the relative Frobenius error in L, as compared to the f-measure. This can be attributed to the fact that even a small error in

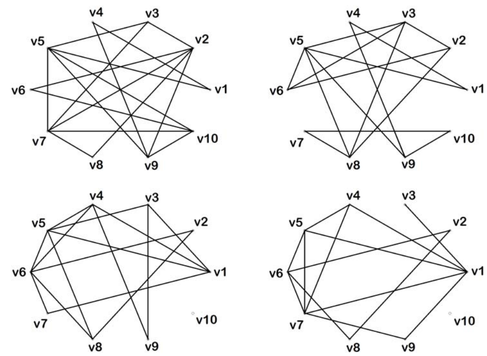

Fig. 1: (left column) Ground truth graphs and (right column) Corresponding estimated graphs. Note that the graphs have been shown as binary(unweighted) graphs.

edge weight will contribute towards the relative error while it is likely not to contribute to the f-measure error.

m	Precision (Ideally 1)	Recall (Ideally 1)	f-measure (Ideally 1)	$\frac{\|\|L-\widehat{L}\|\|_F}{\|\|L\|\|_F}$ (Ideally 0)
1500	0.7692	0.6897	0.7273	0.4
1000	0.6552	0.7037	0.6786	0.55
500	0.55	0.71	0.62	0.64

TABLE I: Graph estimation performance for $N = 10$ and averaged over 100 experiments for each value of m.

B. ANLGBT results

All images used in this section are gray-scale and have 200×200 pixels. We add Gaussian noise with variance (σ) 5 and 10, higher noise variance is typically not observed in practice. The number of similar patches in an image for every patch is fixed to 20. Examples of denoised smooth, natural and texture images using BM3D, NLGBT and ANLGBT algorithm is shown in Figure 2, followed by a few zoomed-in images in Figure 3. An objective evaluation using PSNR, FSIM and SSIM measures for various images is provided in TableII. We have carried out similar experiments with noise variance 10, but do not provide the results here due to lack of space. The performance of ANLGBT is still found to be better or comparative with the best for texture images, but lags behind BM3D in case of natural images by approximately 2 db on average, and by about 4 db for smooth images.

VI. Discussion and Conclusion

In this paper we have presented a framework for learning graph topology, given a collection of graph signals which are sparse in frequency domain. The algorithm does provide a decent solution as far as the f-measure is concerned, but there is room for improvement. A major issue with the algorithm is accuracy of the estimated eigenvector matrix $\widehat{V}$, which is known to be sensitive. Provided with the ground truth eigenvector matrix, our algorithm provides an f-measure between 0.95-0.98.

As an application we use our graph topology estimation algorithm to adapt the NLGBT image denoising procedure on natural and texture images. The ANLGBT algorithm outperforms the NLGBT and BM3D algorithms for noise variance 5, which is a realistic scenario. As of now, the number of similar patches for any patch has been kept to 20, and we believe the performance of the algorithm will improve by including a higher number of similar patches.

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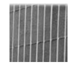
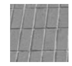
Quality measure	Algorithm						
PSNR	BM3D	35.150	35.555	33.687	33.486	39.602	42.886
	NLGBT	36.778	37.292	34.891	34.766	42.529	**43.993**
	ANLGBT	**36.864**	**37.527**	**36.036**	**35.602**	**42.627**	41.990
FSIM	BM3D	0.9636	0.9626	0.9407	0.9259	0.9831	**0.9924**
	NLGBT	0.9666	0.9687	0.9531	0.9396	0.9846	0.9923
	ANLGBT	**0.9744**	**0.9732**	**0.9730**	**0.9682**	**0.9863**	0.9885
SSIM	BM3D	0.9384	0.9377	0.7936	0.7880	0.9812	0.9892
	NLGBT	0.9533	**0.9554**	0.8476	0.9006	0.9834	**0.9902**
	ANLGBT	**0.9828**	0.9225	**0.9094**	**0.9422**	**0.9845**	0.9848

TABLE II: Performance measures of denoising algorithms for natural, texture and smooth images, with $\sigma = 5$.

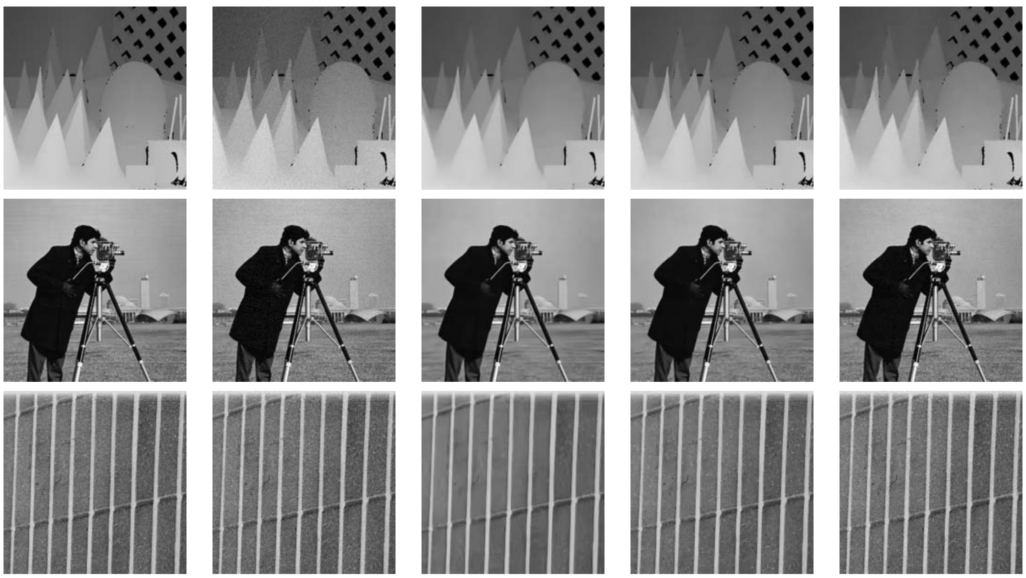


Fig. 2: Denoising examples for a piece-wise smooth, natural and texture image, for $\sigma = 5$. (Column 1) Original image, (Column 2) Noisy image, (Column 3) BM3D results, (Column 4) NLGBT results, (Column 5) ANLGBT results. ANLGBT tends to preserve texture and other details better - observe the grass region and texture of each brick in the cameraman and brick image, respectively. Also refer Figure3.

Fig. 3: Close-up of denoising results of a natural and texture image, with noise variance 5. (Column 1) Original image, (Column 2) Noisy image, (Column 3) BM3D results, (Column 4) NLGBT results, (Column 5) ANLGBT results. Observe the preservation of texture using ANLGBT.