Technical Report Topology Preserving Irregular Image Pyramid (TIIP) February 25, 2022 Darshan Batavia

This technical report is a compilation of the important results from the TIIP algorithm presented in [1]. The goal of the Topology preserving Irregular Image Pyramid (TIIP) algorithm is to build a graph-based irregular image pyramid that preserves the topology and the structure of the image. The top level of the pyramid consists of critical vertices only and the connections between them. The results displayed are the output images that are reconstructed by using the structural information and gray value of the vertices at the top level of the pyramid. Depending on the number critical vertices in the neighbourhood graph of the image, the top level of the pyramid typically consists of 12-35% of vertices at the base level of the pyramid.

Results and Discussion on TIIP algorithm 1

This section displays both the desirable and undesirable results from the TIIP algorithm. The discussion explains the possible reasons for achieving undesirable results. The algorithm was tested on more than 150 images from the Berkley image segmentation data set [3]. This technical report displays 30 examples and their respective reconstructed images (showing both desirable and undesirable results) for comparison. The essential statistical information regrading the construction of the pyramid displays the size of the image, the number of vertices at the base level before the insertion of the hidden saddles, the number of extrema and the number of saddles after contraction of the plateau region and a table showing the required stats at each level of the pyramid. The first column of the table displays the levels of pyramid. For each pyramid level, the second, the third and the fourth column displays the number of surviving slope vertices, the % of surviving vertices and the length of the longest path. The first row of the pyramid level states the level at which all the plateau regions were collapsed to a single surviving vertex. In other words, all edges e with contrast c(e) = 0 are contracted and the edges with non-zero contrast are surviving.

Since the critical vertices are preserved, the number of extrema and saddles remain constant throughout the pyramid after the contraction of the plateau region. The second column displays the number of surviving slope vertices while the third column shows the percentage of surviving vertices (including the extrema, the saddles and the slope vertices). It can be clearly seen that the two consecutive entries in column two and three are exactly the same. The first entry correspond to the contraction operation and the later entry correspond to the removal operation. The consecutive entries are same due to the fact that the number of vertices remain unchanged by the edge removal operation that follows after the edge contraction operation.

The following table assists in easy navigation through the results:

% surviving vertices	Image id as per [3]
12-15%	43074 1, 160068 2, 210088 3, 106024 4, 69015 5
16-20%	14037 6, 101087 7, 108005 8, 108082 9, 69040 10, 208001 11, 253027 12,86000 13
21-25%	38092 14, 102061 15, 78004 16, 19021 17, 197017 18, 65033 19, 97033 20, 300091 21, 296059 22, 148026 23, 38082 24
26-35%	41069 25, 55073 26, 156065 27, 101085 28, 37073 29, 86016 30

Image 43074 [3]



(a) Original image



(b) Reconstructed image from 12.9% of vertices at the top level of the pyramid.

Figure 1: Original and Reconstructed image for Image 43074

Essential Statistics for image 43074 [3]

Size of the image $= 481 \times 321$ Total number of vertices = 154401Number of extrema = 14842Number of saddles = 4946

pyramid level	slope vertices	% of surviving vertices	longest path
23	88045	69,96652	242
24	39331	$38,\!41491$	291
25	39331	$38,\!41491$	291
26	15656	$23,\!07951$	313
27	15656	$23,\!07951$	313
28	5619	$16,\!57891$	313
29	5619	$16,\!57891$	313
30	1801	$14,\!10548$	333
31	1801	$14,\!10548$	333
32	501	$13,\!26352$	333
32	501	13,26352	333
33	107	$13,\!00834$	451
33	107	$13,\!00834$	451
34	11	12,94616	451
34	11	12,94616	451
35	3	$12,\!94098$	538
35	3	$12,\!94098$	538
36	0	12,93904	538

Table 1: Add caption

Discussion: The texture information of the grass in front of the bird and the feathers of the bird are clearly preserved, while the background is blurred.

Image 160068 [3]



(a) Original image



(b) Reconstructed image from 13.6% of vertices at the top level of the pyramid.

Figure 2: Original and Reconstructed image for Image 160068

Essential Statistics for image 160068 [3]

Size of the image $= 481 \times 321$ Total number of vertices = 154401Number of extrema = 15952Number of saddles = 4589

pyramid level	slope vertices	% of surviving vertices	longest path
29	61469	53,46015	943
30	25873	30,39488	1039
31	25873	$30,\!39488$	1039
32	9130	19,54845	1106
33	9130	19,54845	1106
34	2673	$15,\!36583$	1139
35	2673	$15,\!36583$	1139
36	712	14,09447	1139
37	712	14,09447	1139
38	176	13,74732	1139
38	176	13,74732	1139
39	46	13,66312	1139
39	46	$13,\!66312$	1139
40	8	13,63851	1139
40	8	13,63851	1139
41	2	13,63463	1139
41	2	$13,\!63463$	1139
42	0	$13,\!63333$	1139

Table 2: Add caption

Discussion: The hairs on the face of the leopard and the texture on the leopard's body is preserved. Artifacts can be observed in the background especially on the bottom right corner.

Image 210088 [3]



(a) Original image



(b) Reconstructed image from 11.88% of vertices at the top level of the pyramid.

Figure 3: Original and Reconstructed image for Image 210088

Essential Statistics for image 210088 [3]

Size of the image $= 321 \times 481$ Total number of vertices = 154401Number of extrema = 14067Number of saddles = 3952

pyramid level	slope vertices	% of surviving vertices	longest path
20	89988	70,17571	54
21	42203	$39,\!22513$	93
22	42203	39,22513	93
23	17885	$23,\!4707$	131
24	17885	$23,\!4707$	131
25	6932	$16,\!37554$	138
26	6932	16,37554	138
27	2375	$13,\!42414$	149
28	2375	13,42414	149
29	742	$12,\!36585$	220
30	742	$12,\!36585$	220
31	185	12,0051	249
32	185	12,0051	249
33	43	11,91314	305
34	43	11,91314	305
35	9	11,89111	318
35	9	11,89111	318
36	0	11,88529	320

Table 3: Add caption

Discussion: The pastelization effect is clearly visible in this image on the fish as well as in the background. Besides, due to the smooth texture of this image, only 11.8% of vertices contribute as critical vertices. The boundary of the object in the reconstructed image is not crisp as it appears in the original image. I predict that this is because the implementation of the TIIP algorithm is performed after converting the RGB image to grey scale. This leads to loss of information especially for the vertices that act as a border between two objects.

Image 106024 [3]



(a) Original image



(b) Reconstructed image from 12.5% of vertices at the top level of the pyramid.

Figure 4: Original and Reconstructed image for Image 106024

Essential Statistics for image 106024 [3]

Size of the image $= 481 \times 321$ Total number of vertices = 154401Number of extrema = 14142Number of saddles = 4860

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pyramid level	slope vertices	% of surviving vertices	longest path
29	80774	64,82147	842
30	36295	$36,\!01078$	921
31	36295	36,01078	921
32	15037	22,24208	947
33	15037	22,24208	947
34	6061	$16,\!42865$	997
35	6061	$16,\!42865$	997
36	2369	$14,\!03747$	1063
37	2369	$14,\!03747$	1063
38	888	$13,\!07828$	1164
38	888	$13,\!07828$	1164
39	301	12,6981	1165
39	301	$12,\!6981$	1165
40	92	$12,\!56274$	1288
40	92	$12,\!56274$	1288
41	24	12,5187	1289
41	24	12,5187	1289
42	5	12,5064	1325
42	5	$12,\!5064$	1325
43	0	12,50316	1325

Table 4: Add caption

Discussion: The texture information in the foreground object (penguin and the rocks) is preserved. A clear pastelization effect can be observed in the background where the objects are not blurred and not differentiable in the original image as well.

Image 69015 [3]





(b) Reconstructed image from 15.79% of vertices at the top level of the pyramid.

Figure 5: Original and Reconstructed image for Image 69015

Essential Statistics for image 69015 [3]

Size of the image $= 321 \times 481$ Total number of vertices = 154401Number of extrema = 18909Number of saddles = 5308

pyramid level	slope vertices	% of surviving vertices	longest path
35	95533	$77,\!66789$	6559
36	41997	$42,\!99454$	6701
37	41997	$42,\!99454$	6701
38	15470	$25,\!81266$	6752
39	15470	$25,\!81266$	6752
40	4874	$18,\!94936$	6782
41	4874	$18,\!94936$	6782
42	1322	$16,\!64821$	6810
43	1322	$16,\!64821$	6810
44	313	$15,\!99407$	6810
45	313	$15,\!99407$	6810
46	69	$15,\!83539$	6810
46	69	$15,\!83539$	6810
47	20	$15,\!80365$	6810
47	20	$15,\!80365$	6810
48	6	15,79459	6810
48	6	15,79459	6810
49	2	15,792	6810
49	2	15,792	6810
50	1	15,79135	6810
50	1	15,79135	6810
51	0	15,7907	6810

Table 5: Add caption

Discussion: This is one of the good result with a near perfect reconstruction with only 15.7% of information.

Image 14037 [3]



(a) Original image



(b) Reconstructed image from 16.1% of vertices at the top level of the pyramid.

Figure 6: Original and Reconstructed image for Image 14037

Essential Statistics for image 14037 [3]

Size of the image $= 481 \times 321$ Total number of vertices = 154401Number of extrema = 18551Number of saddles = 5812

pyramid level	slope vertices	% of surviving vertices	longest path
39	59173	54,43294	20153
40	22603	30,74073	20475
41	22603	30,74073	20475
42	7083	$20,\!68834$	20598
43	7083	$20,\!68834$	20598
44	1802	17,26802	20636
45	1802	17,26802	20636
46	396	$16,\!35741$	20636
46	396	$16,\!35741$	20636
47	79	16,1521	20636
47	79	16,1521	20636
48	12	16,1087	20636
48	12	16,1087	20636
49	5	$16,\!10417$	20636
49	5	$16,\!10417$	20636
50	2	16,10223	20636
50	2	16,10223	20636
51	0	16,10093	20636

Table 6: Add caption

Discussion: Pastelization of colours especially on the clouds, but the reconstructed image is constructed form 16.1% of the vertices only. Minute details such as hands of the person are also preserved by the reconstructed image.

Image 101087 [3]





(b) Reconstructed image from 16.54% of vertices at the top level of the pyramid.

Figure 7: Original and Reconstructed image for Image 101087

Essential Statistics for image 101087 [3]

Size of the image $= 321 \times 481$ Total number of vertices = 154401Number of extrema = 19283Number of saddles = 5934

pyramid level	slope vertices	% of surviving vertices	longest path
45	61393	$56,\!30533$	32728
46	23614	$31,\!83464$	33157
47	23614	$31,\!83464$	33157
48	7144	21,16696	33358
49	7144	21,16696	33358
50	1636	$17,\!59963$	33463
51	1636	$17,\!59963$	33463
52	295	16,73111	33481
53	295	16,73111	33481
54	46	$16,\!56984$	33489
54	46	16,56984	33489
55	5	$16,\!54329$	33491
55	5	$16{,}54329$	33491
56	0	$16,\!54005$	33491

Table 7: Add caption

Discussion: A difference can be noticed in the outfit of the person in the original image and the reconstructed image. The reconstructed image appears to contain only the higher frequency information by eliminating the lower frequency information. The minute details of the reflection on the waves are well preserved.

Image 108005 [3]



(a) Original image



(b) Reconstructed image from 16.77% of vertices at the top level of the pyramid.

Figure 8: Original and Reconstructed image for Image 108005

Essential Statistics for image 108005 [3]

Size of the image $= 481 \times 321$ Total number of vertices = 154401Number of extrema = 19484Number of saddles = 6325

pyramid level	slope vertices	% of surviving vertices	longest path
15	109201	87,50008	39
16	48783	48,36886	47
17	48783	48,36886	47
18	18240	$28,\!5866$	57
19	18240	$28,\!5866$	57
20	5770	20,50958	73
21	5770	20,50958	73
22	1528	17,76154	97
22	1528	17,76154	97
23	338	$16,\!99082$	136
24	338	16,99082	136
25	73	$16,\!81919$	259
25	73	$16,\!81919$	259
26	19	16,78422	272
26	19	16,78422	272
27	2	16,77321	420
27	2	16,77321	420
28	0	16,77191	421

Table 8: Add caption

Discussion: The features and hairs on the face of the tiger in the image are well preserved. The artifacts can be observed on the border of the strips over the body of the tiger. I predict that loss of information while converting the RGB image to grey scale is a key reason for this artifact.

Image 108082 [3]



(a) Original image



(b) Reconstructed image from ${\bf 16.8\%}$ of vertices at the top level of the pyramid.

Figure 9: Original and Reconstructed image for Image 108082

Essential Statistics for image 108082 [3]

Size of the image $= 481 \times 321$ Total number of vertices = 154401Number of extrema = 20313Number of saddles = 5391

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pyramid level	slope vertices	% of surviving vertices	longest path
36	86506	72,83502	6951
37	37331	40,98419	7442
38	37331	40,98419	7442
39	13704	$25,\!67989$	7514
40	13704	$25,\!67924$	7514
41	4315	$19,\!59767$	7554
42	4315	$19,\!59767$	7554
43	1209	$17,\!58538$	7592
43	1209	$17,\!58538$	7592
44	295	16,99341	7602
44	295	$16,\!99341$	7602
45	52	16,83603	7629
45	52	$16,\!83603$	7629
46	5	$16,\!80559$	7629
46	5	$16,\!80559$	7629
47	0	$16,\!80235$	7629

Table 9: Add caption

Discussion: The features and hairs on the face of the tiger in the image are well preserved. The artifacts can be observed on the border of the strips over the body of the tiger. I predict that loss of information while converting the RGB image to grey scale is a key reason for this artifact.

Image 69040 [3]



(a) Original image



(b) Reconstructed image from 18.7% of vertices at the top level of the pyramid.

Figure 10: Original and Reconstructed image for Image 69040

Essential Statistics for image 69040 [3]

Size of the image $= 481 \times 321$ Total number of vertices = 154401Number of extrema = 21948Number of saddles = 6858

pyramid level	slope vertices	% of surviving vertices	longest path
12	110646	90,36664	19
13	48494	$50,\!11302$	24
14	48494	$50,\!11302$	24
15	17941	30,32493	41
16	17941	$30,\!32493$	41
17	5529	$22,\!28548$	58
18	5529	22,28548	58
19	1359	$19,\!58407$	73
20	1359	$19,\!58407$	73
21	278	$18,\!88395$	123
21	278	$18,\!88395$	123
22	37	18,72786	131
22	37	18,72786	131
23	4	18,70649	226
23	4	18,70649	226
24	0	18,70389	227

Table 10: Add caption

Discussion: Even though the colour of the dried plant and the kangaroo are almost similar, the dried plans in occluding the kangaroo's body are distinguishable in the reconstructed image.

Image 208001 [3]



(a) Original image



(b) Reconstructed image from $\mathbf{19.44\%}$ of vertices at the top level of the pyramid.

Figure 11: Original and Reconstructed image for Image 208001

Essential Statistics for image 208001 [3]

Size of the image $= 321 \times 481$ Total number of vertices = 154401Number of extrema = 23222Number of saddles = 6555

pyramid level	slope vertices	% of surviving vertices	longest path
22	100700	84,66331	237
23	42614	47,04244	267
24	42614	47,04244	267
25	14546	28,86316	276
26	14546	28,86316	276
27	3839	21,92861	283
28	3839	21,92861	283
29	764	$19,\!9351$	283
30	764	$19,\!9351$	283
31	135	19,52772	283
31	135	$19,\!52772$	283
32	17	$19,\!4513$	283
32	17	$19,\!4513$	283
33	4	$19,\!44288$	283
33	4	$19,\!44288$	283
34	0	$19,\!44029$	283

Table 11: Add caption

Discussion: The fine texture on the mushroom and the leaves in the background are reconstructed with a near perfect accuracy. There is still space for improvement considering the non-crisp boundary of the mushroom stem.

Image 253027 [3]



(a) Original image



(b) Reconstructed image from 19.96% of vertices at the top level of the pyramid.

Figure 12: Original and Reconstructed image for Image 253027

Essential Statistics for image 253027 [3]

Size of the image $= 481 \times 321$ Total number of vertices = 154401Number of extrema = 24307Number of saddles = 6387

pyramid level	slope vertices	% of surviving vertices	longest path
18	97734	83,26954	38
19	41363	46,76006	54
20	41363	46,76006	54
21	14615	$29,\!4331$	58
22	14615	$29,\!4331$	58
23	4614	22,95581	107
24	4614	22,95581	107
25	1393	20,86968	127
26	1393	20,86968	127
27	400	$20,\!22655$	149
28	400	$20,\!22655$	149
29	91	20,02642	250
30	91	20,02642	250
31	14	$19,\!97655$	381
32	14	$19,\!97655$	381
33	5	19,97073	381
34	5	19,97073	381
35	2	19,96878	381
35	2	19,96878	381
36	0	19,96749	381

Table 12: Add caption

Image 86000 [3]



(a) Original image



(b) Reconstructed image from 20.06% of vertices at the top level of the pyramid.

Figure 13: Original and Reconstructed image for Image 86000

Essential Statistics for image 86000 [3]

Size of the image $= 321 \times 481$ Total number of vertices = 154401Number of extrema = 24408Number of saddles = 6016

$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	pyramid level	slope vertices	% of surviving vertices	longest path
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	29	74414	68,2664	308
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$				
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	30	29191	$38,\!9706$	351
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	31	29191	$38,\!9706$	351
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	32	9459	26,1896	355
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	33	9459	$26,\!1896$	355
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	34	2526	$21,\!69675$	355
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	35	2526	$21,\!69675$	355
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	36	634	$20,\!47072$	355
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	37	634	$20,\!47072$	355
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	38	150	$20,\!15725$	355
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	38	150	20,15725	355
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	39	40	20,08601	380
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	39	40	20,08601	380
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	40	12	20,06788	385
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	40	12	20,06788	385
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	41	3	20,06205	387
42 0 20,0601 387	41	3	20,06205	387
	42	0	20,0601	387

Table 13: Add caption

Image 38092 [3]



(a) Original image



(b) Reconstructed image from 22.24% of vertices at the top level of the pyramid.

Figure 14: Original and Reconstructed image for Image 38092

Essential Statistics for image 38092 [3]

Size of the image $= 481 \times 321$ Total number of vertices = 154401Number of extrema = 26392Number of saddles = 7539

pyramid level	slope vertices	% of surviving vertices	longest path
27	85253	77,46323	524
28	33419	$43,\!88767$	550
29	33419	$43,\!88767$	550
30	10500	29,04256	584
31	10500	29,04256	584
32	2601	$23,\!92601$	600
33	2601	$23,\!92601$	600
34	534	22,58729	600
35	534	$22,\!58729$	600
36	149	$22,\!33794$	600
37	149	$22,\!33794$	600
38	39	$22,\!2667$	600
38	39	22,2667	600
39	12	22,24921	600
40	12	22,24921	600
41	2	$22,\!24273$	600
41	2	$22,\!24273$	600
42	0	22,24144	600

Table 14: Add caption

Image 102061 [3]



(a) Original image



(b) Reconstructed image from 22.3% of vertices at the top level of the pyramid.

Figure 15: Original and Reconstructed image for Image 102061

Essential Statistics for image 102061 [3]

Size of the image $= 321 \times 481$ Total number of vertices = 154401Number of extrema = 24786Number of saddles = 9106

pyramid level	slope vertices	% of surviving vertices	longest path
27	69802	67,52871	217
28	25329	38,71737	267
29	25329	38,71737	267
30	7433	$27,\!12612$	269
31	7433	27,12612	269
32	1788	$23,\!46941$	269
33	1788	$23,\!46941$	269
34	363	$22,\!54649$	269
35	363	$22,\!54649$	269
36	73	$22,\!35802$	269
37	73	$22,\!35802$	269
38	15	$22,\!32045$	269
38	15	$22,\!32045$	269
39	1	22,31138	269
40	1	22,31138	269
41	0	22,31074	269

Table 15: Add caption

Discussion: The minute details like the spire over the roof and the fine lines on the furnace exhaust are preserved.

Image 78004 [3]



(b) Reconstructed image from 22.6% of vertices at the top level of the pyramid.

Figure 16: Original and Reconstructed image for Image 78004

Essential Statistics for image 78004 [3]

Size of the image $= 321 \times 481$ Total number of vertices = 154401Number of extrema = 24801Number of saddles = 9777

pyramid level	slope vertices	% of surviving vertices	longest path
38	84241	77,19056	2618
39	32588	43,73612	2675
40	32588	43,73612	2675
41	10470	$29,\!41108$	2707
42	10470	29,41108	2707
43	2591	24,30749	2729
44	2591	24,30749	2729
45	555	$22,\!98884$	2741
46	555	$22,\!98884$	2741
47	105	$22,\!69739$	2741
47	105	$22,\!69739$	2741
48	16	22,6391	2741
48	16	22,6391	2741
49	2	22,63003	2741
49	2	$22,\!63003$	2741
50	0	$22,\!62874$	2768

Table 16: Add caption

Image 19021 [3]



(a) Original image



(b) Reconstructed image from 22.35% of vertices at the top level of the pyramid.

Figure 17: Original and Reconstructed image for Image 19021

Essential Statistics for image 19021 [3]

Size of the image $= 481 \times 321$ Total number of vertices = 154401Number of extrema = 27027Number of saddles = 7152

pyramid level	slope vertices	% of surviving vertices	longest path
24	87474	79,01115	575
25	34461	$44,\!67393$	589
26	34461	$44,\!67393$	589
27	10538	$29,\!17986$	591
28	10538	$29,\!17986$	591
29	2358	$23,\!88197$	591
30	2358	$23,\!88197$	591
31	399	22,6132	591
31	399	22,6132	591
32	58	$22,\!39234$	591
32	58	22,39234	591
33	8	$22,\!35996$	591
33	8	$22,\!35996$	591
34	2	$22,\!35607$	591
34	2	$22,\!35607$	591
35	1	$22,\!35543$	591
35	1	$22,\!35543$	591
36	0	$22,\!35478$	591

Table 17: Add caption

Image 197017 [3]



(a) Original image



(b) Reconstructed image from 23.86% of vertices at the top level of the pyramid.

Figure 18: Original and Reconstructed image for Image 197017

Essential Statistics for image 197017 [3]

Size of the image $= 481 \times 321$ Total number of vertices = 154401Number of extrema = 27421Number of saddles = 9127

pyramid level	slope vertices	% of surviving vertices	longest path
16	85569	79,28511	39
17	32596	44,97574	49
18	32596	44,97574	49
19	9931	$30,\!29449$	58
20	9931	$30,\!29449$	58
21	2479	$25,\!46745$	76
22	2479	$25,\!46745$	76
23	585	$24,\!24078$	104
24	585	$24,\!24078$	104
25	126	$23,\!9435$	127
25	126	$23,\!9435$	127
26	17	$23,\!8729$	154
27	17	$23,\!8729$	154
28	5	$23,\!86513$	188
28	5	$23,\!86513$	188
29	2	23,86319	191
29	2	$23,\!86319$	191
30	1	$23,\!86254$	191
30	1	$23,\!86254$	191
31	0	$23,\!86189$	191

Table 18: Add caption

Discussion: Fine details like the shadow of the metal wire on the ground and small flowers in front of the horses are preserved in the reconstructed image.

Image 65033 [3]



(a) Original image



(b) Reconstructed image from 23.9% of vertices at the top level of the pyramid.

Figure 19: Original and Reconstructed image for Image 65033

Essential Statistics for image 65033 [3]

Size of the image $= 481 \times 321$ Total number of vertices = 154401Number of extrema = 28665Number of saddles = 8203

pyramid level	slope vertices	% of surviving vertices	longest path
31	101605	89,78374	898
32	40783	$50,\!39151$	946
33	40783	50,39151	946
34	12892	$32,\!32686$	953
35	12892	$32,\!32686$	953
36	3101	$25,\!98494$	957
37	3101	$25,\!98494$	957
38	566	$24,\!34311$	968
39	566	$24,\!34311$	968
40	85	$24,\!03158$	969
41	85	24,03158	969
42	12	$23,\!9843$	969
42	12	$23,\!9843$	969
43	2	$23,\!97782$	969
43	2	$23,\!97782$	969
44	0	$23,\!97653$	969

Table 19: Add caption

Image 97033 [3]



(a) Original image



(b) Reconstructed image from 24.2% of vertices at the top level of the pyramid.

Figure 20: Original and Reconstructed image for Image 97033

Essential Statistics for image 97033 [3]

Size of the image $= 481 \times 321$ Total number of vertices = 154401Number of extrema = 28164Number of saddles = 8884

pyramid level	slope vertices	% of surviving vertices	longest path
20	88601	81,59727	109
21	34437	46,51524	120
22	34437	46,51524	120
23	10288	30,87286	126
24	10288	30,87286	126
25	2240	$25,\!65916$	126
26	2240	$25,\!65916$	126
27	387	$24,\!4571$	126
28	387	$24,\!4571$	126
29	71	$24,\!25243$	126
29	71	24,25243	126
30	16	24,21681	126
30	16	24,21681	126
31	5	24,20969	126
31	5	24,20969	126
32	1	24,2071	126
32	1	24,2071	126
33	0	24,20645	126
	1		1

Table 20: Add caption

Image 300091 [3]



(a) Original image



(b) Reconstructed image from 24.31% of vertices at the top level of the pyramid.

Figure 21: Original and Reconstructed image for Image 300091

Essential Statistics for image 300091 [3]

Size of the image $= 481 \times 321$ Total number of vertices = 154401Number of extrema = 28710Number of saddles = 8288

pyramid level	slope vertices	% of surviving vertices	longest path
20	75820	73,4231	109
21	26694	41,60336	115
22	26694	$41,\!60336$	115
23	6814	28,72715	126
24	6814	28,72715	126
25	1223	$25,\!10606$	134
26	1223	$25,\!10606$	134
27	167	$24,\!42147$	245
27	167	$24,\!42147$	245
28	27	$24,\!3308$	246
28	27	$24,\!3308$	246
29	4	$24,\!3159$	248
29	4	$24,\!3159$	248
30	1	$24,\!31396$	248
30	1	$24,\!31396$	248
31	0	24,31331	248

Table 21: Add caption

Image 296059 [3]



(a) Original image



(b) Reconstructed image 296059 from 24.42% of vertices at the top level of the pyramid.

Figure 22: Original and Reconstructed image for Image 296059

Essential Statistics for image 296059 [3]

Size of the image $= 481 \times 321$ Total number of vertices = 154401Number of extrema = 29335Number of saddles = 7892

pyramid level	slope vertices	% of surviving vertices	longest path
23	72730	$71,\!54228$	146
24	25497	40,94404	239
25	25497	40,94404	239
26	6711	28,77637	274
27	6711	28,77637	274
28	1360	$25,\!31007$	274
28	1360	$25,\!31007$	274
29	252	$24,\!59246$	274
30	252	$24,\!59246$	274
31	44	$24,\!45774$	274
31	44	$24,\!45774$	274
32	12	$24,\!43702$	274
32	12	$24,\!43702$	274
33	4	$24,\!43184$	274
33	4	$24,\!43184$	274
34	2	$24,\!43054$	274
34	2	$24,\!43054$	274
35	1	$24,\!42989$	274
35	1	$24,\!42989$	274
36	0	$24,\!42925$	274

Table 22: Add caption

Image 148026 [3]



(b) Reconstructed image from 25.07% of vertices at the top level of the pyramid.

Figure 23: Original and Reconstructed image for Image 148026

Essential Statistics for image 148026 [3]

Size of the image $= 321 \times 481$ Total number of vertices = 154401Number of extrema = 30104Number of saddles = 8357

pyramid level	slope vertices	% of surviving vertices	longest path
31	94381	86,20475	3442
32	37436	49,32351	3627
33	37436	49,32351	3627
34	11427	$32,\!47777$	3682
35	11427	$32,\!47777$	3682
36	2515	26,70449	3730
37	2515	26,70449	3730
38	402	$25,\!33533$	3742
38	402	$25,\!33533$	3742
39	44	$25,\!10346$	3742
39	44	$25,\!10346$	3742
40	4	$25,\!07756$	3742
40	4	$25,\!07756$	3742
41	1	$25,\!07561$	3742
41	1	$25,\!07561$	3742
42	0	25,07497	3742

Table 23: Add caption

Image 38082 [3]



(a) Original image



(b) Reconstructed image from 25.7% of vertices at the top level of the pyramid.

Figure 24: Original and Reconstructed image for Image 38082

Essential Statistics for image 38082 [3]

Size of the image $= 481 \times 321$ Total number of vertices = 154401Number of extrema = 30419Number of saddles = 9195

pyramid level	slope vertices	% of surviving vertices	longest path
21	103381	$92,\!67427$	190
22	40127	51,70692	217
23	40127	51,70692	217
24	11777	$33,\!34499$	220
25	11777	$33,\!34499$	220
26	2371	$27,\!25241$	220
27	2371	$27,\!25241$	220
28	297	$25,\!90851$	220
28	297	$25,\!90851$	220
29	36	25,73947	220
29	36	25,73947	220
30	4	25,71875	220
30	4	25,71875	220
31	0	25,71615	220

Table 24: Add caption

Image 41069 [3]



(a) Original image



(b) Reconstructed image from 26.5% of vertices at the top level of the pyramid.

Figure 25: Original and Reconstructed image for Image 41069

Essential Statistics for image 41069 [3]

Size of the image $= 481 \times 321$ Total number of vertices = 154401Number of extrema = 31885Number of saddles = 8961

pyramid level	slope vertices	% of surviving vertices	longest path
13	104092	93,92102	26
14	41069	$53,\!10328$	34
15	41069	$53,\!10328$	34
16	11845	34,17594	34
17	11845	34,17594	34
18	2358	28,03091	46
19	2358	28,03091	46
20	334	26,72003	58
21	334	26,72003	58
22	36	$26,\!52703$	85
23	36	26,52703	85
24	4	26,50631	85
24	4	$26,\!50631$	85
25	0	26,50371	85

Table 25: Add caption

Image 55073 [3]



(b) Reconstructed image from 29% of vertices at the top level of the pyramid.

Figure 26: Original and Reconstructed image for Image 55073

Essential Statistics for image 55073 [3]

Size of the image $= 321 \times 481$ Total number of vertices = 154401Number of extrema = 35565Number of saddles = 9190

pyramid level	slope vertices	% of surviving vertices	longest path
13	100002	93,83294	52
14	37585	$53,\!40769$	55
15	37585	$53,\!40769$	55
16	9991	$35,\!53539$	55
17	9991	$35{,}53539$	55
18	1747	$30,\!1954$	61
19	1747	$30,\!1954$	61
20	203	$29,\!19476$	61
20	203	$29,\!19476$	61
21	28	29,08142	61
21	28	29,08142	61
22	6	29,06717	61
22	6	29,06717	61
23	3	29,06523	61
23	3	29,06523	61
24	0	29,06328	61

Table 26: Add caption

Discussion: It is a near perfect reconstruction and it is hard to distinguish between the original and the reconstructed image. Since the image contains a lot of texture information on the rocks, the number of critical vertices are considerably higher (at 29%) as compared to the previous images. This improved the quality of the reconstruction as well. Though the quality can be compromised by reducing the insignificant critical vertices with the help of homology persistence. This work will be continued in the near future.

Image 156065 [3]



(a) Original image



(b) Reconstructed image from 29.86% of vertices at the top level of the pyramid.

Figure 27: Original and Reconstructed image for Image 156065

Essential Statistics for image 156065 [3]

Size of the image $= 481 \times 321$ Total number of vertices = 154401Number of extrema = 36997Number of saddles = 8918

pyramid level	slope vertices	% of surviving vertices	longest path
31	91504	89,13155	2497
32	33471	$51,\!54565$	2655
33	33471	$51,\!54565$	2655
34	9119	35,77308	2696
35	9119	35,77308	2696
36	1842	31,06003	2706
37	1842	$31,\!06003$	2706
38	314	30,0704	2706
39	314	30,0704	2706
40	61	$29,\!90589$	2706
40	61	29,90589	2706
41	10	29,87286	2706
41	10	29,87286	2706
42	3	29,86833	2706
42	3	29,86833	2706
43	1	29,86703	2706
43	1	29,86703	2706
44	0	29,86639	2706

Table 27: Add caption

Image 101085 [3]



(b) Reconstructed image from 30.52% of vertices at the top level of the pyramid.

Figure 28: Original and Reconstructed image for Image 101085

Essential Statistics for image 101085 [3]

Size of the image $= 321 \times 481$ Total number of vertices = 154401Number of extrema = 37279Number of saddles = 9727

pyramid level	slope vertices	% of surviving vertices	longest path
30	95381	92,29863	1824
31	34909	$53,\!13243$	1910
32	34909	$53,\!13243$	1910
33	9179	$36,\!46738$	1961
34	9179	$36,\!46738$	1961
35	1590	$31,\!55161$	1969
36	1590	$31,\!55161$	1969
37	205	$30,\!65459$	1969
38	205	$30,\!65459$	1969
39	16	$30,\!53219$	1979
39	16	$30,\!53219$	1979
40	0	$30,\!52182$	1980

Table 28: Add caption

Image 37073 [3]



(a) Original image



(b) Reconstructed image from 30.9% of vertices at the top level of the pyramid.

Figure 29: Original and Reconstructed image for Image 37073

Essential Statistics for image 37073 [3]

Size of the image $= 481 \times 321$ Total number of vertices = 154401Number of extrema = 36799Number of saddles = 10512

pyramid level	slope vertices	% of surviving vertices	longest path
28	80606	83,13741	1421
29	26180	47,88764	1484
30	26180	47,88764	1484
31	5927	34,7705	1520
32	5927	34,7705	1520
33	1019	$31,\!59176$	1531
34	1019	$31,\!59176$	1531
35	166	31,03866	1531
35	166	31,03866	1531
36	34	$30,\!95317$	1531
36	34	30,95317	1531
37	7	$30,\!93568$	1531
37	7	$30,\!93568$	1531
38	0	30,93115	1531

Table 29: Add caption

Image 86016 [3]





(b) Reconstructed image from 35.8% of vertices at the top level of the pyramid.

Figure 30: Original and Reconstructed image for Image 86016

Essential Statistics for image 86016 [3]

Size of the image $= 481 \times 321$ Total number of vertices = 154401Number of extrema = 44493Number of saddles = 10793

pyramid level	slope vertices	% of surviving vertices	longest path
7	94310	96,91971	5
8	32577	56,93746	9
9	32577	56,93746	9
10	7392	40,62603	16
11	7392	40,62603	16
12	1046	$36{,}51596$	29
13	1046	$36{,}51596$	29
14	108	$35,\!90845$	29
14	108	$35,\!90845$	29
15	11	$35,\!84562$	29
15	11	$35,\!84562$	29
16	1	$35,\!83915$	29
16	1	$35,\!83915$	29
17	0	$35,\!8385$	29

Table 30: Add caption

References

- [1] Darshan Batavia, Rocio Gonzalez-Diaz, and Walter G Kropatsch. Image = structure + few colors. In Structural, Syntactic, and Statistical Pattern Recognition - Joint IAPR International Workshops, S+SSPR 2020, Padua, Italy, January 21-22, 2021, Proceedings, volume 12644 of Lecture Notes in Computer Science, pages 365–375. Springer, 2020.
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- [3] David Martin, Charless Fowlkes, Doron Tal, and Jitendra Malik. A database of human segmented natural images and its application to evaluating segmentation algorithms and measuring ecological statistics. In *Computer Vision, 2001. ICCV 2001. Proceedings. Eighth IEEE International Conference on*, volume 2, pages 416–423. IEEE, 2001.