Approximating TSP solution by Simplifying the Input with Graph Pyramids

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The traveling salesperson (TSP) finds the shortest tour through n cities. It is known that this problem is difficult to solve when the number of cities is large, in fact IT is NP-hard. In spite of this, there exist configurations of cities where a trivial closest neighbor connecting algorithm finds the optimal solution. Instead of finding the solution of the input with a large number of cities, the problem is first approximated into a simpler form containing smaller number of cities, which is then solved optimally. Pyramid solution strategies in a bottom-up way convert a 2D Euclidean TSP problem with a large number of cities into successively smaller problems with similar layout and solution until the number of cities is small enough to seek the optimal solution. Expanding this solution in a top-down manner to the lower levels of the pyramid approximates the solution. Regular pyramids lack shift invariance, thus by shifting the input different solutions are produced. An irregular dual graph pyramid adapts its structure to the data and is shift invariant. It is known that the length of the minimal spanning tree (MST) is lower bound of the length of the shortest TSP tour. Christofiedes (1976) used MST to produce an approximating TSP solution with the upper bound 3/2 shortest TSP tour. A version of Boruvca's MST construction will be applied to the solution of the TSP problem. Inserting further cities along the optimal tour does not change its length. This increases the size of a given TSP problem into a large class where the trivial algorithm finds the original optimal solution. This may be of interest for two reasons: 1) it allows the generation of a large variety of new problems with a known optimal solution. 2) The difficulty of a given TSP problem could be related to the density of cities along the optimal tour. A closest point algorithm will be applied to generate classes of new TSP problem with known solution.

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