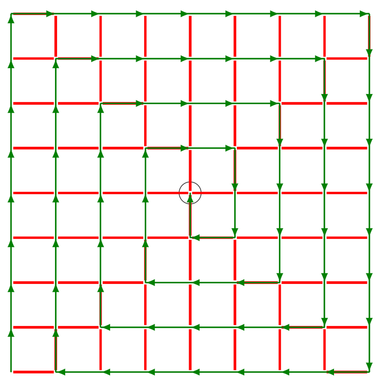

Discrete Spiral and Max-Link, Complementary Tools for Vision

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PRIP from 1990 - 2021
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Image

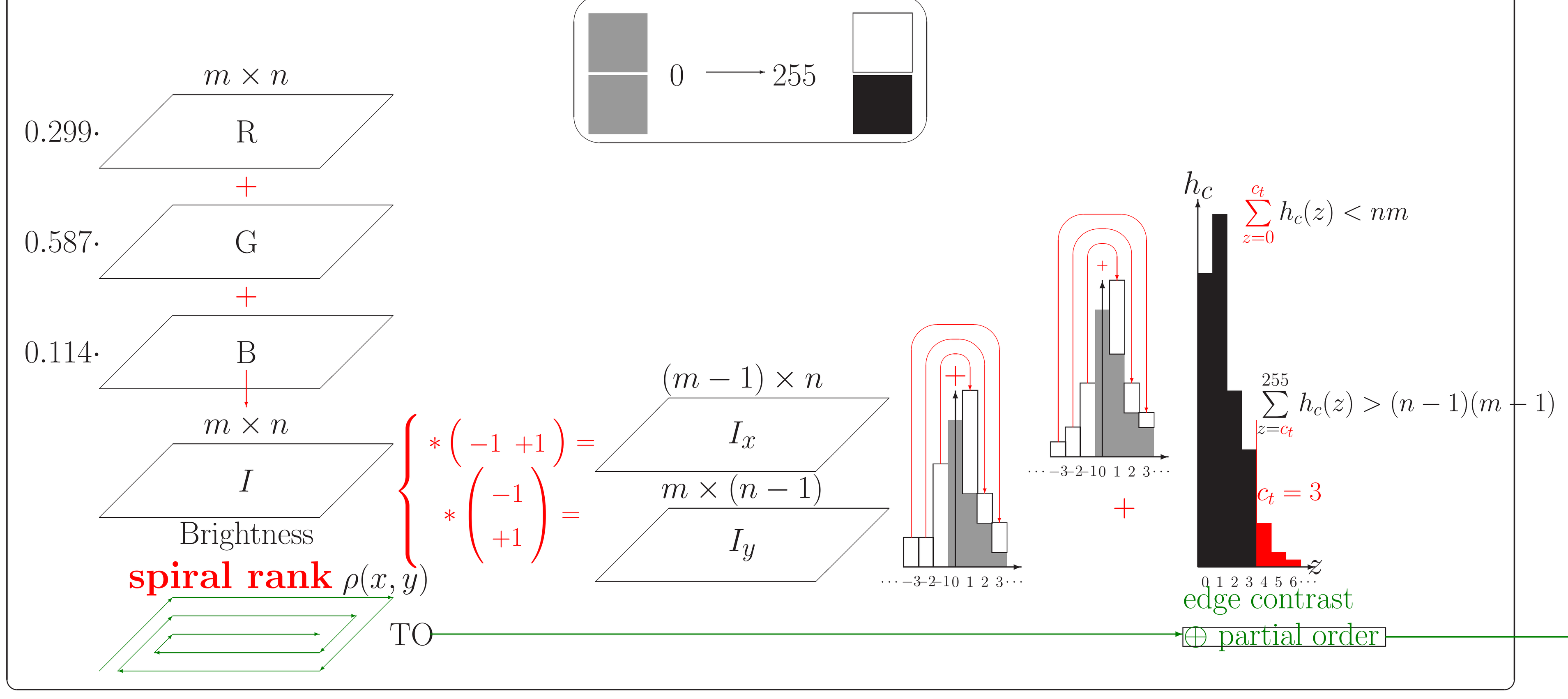
$I(m \times n)$

Contrast

$c : I \mapsto [0, 255]$

Contrast Histogram [3]

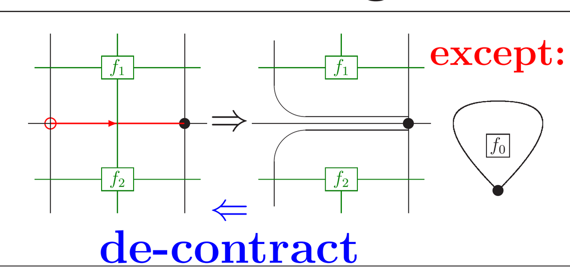
$h_c(z) = \#\{c(e) = z | e \in E\}$

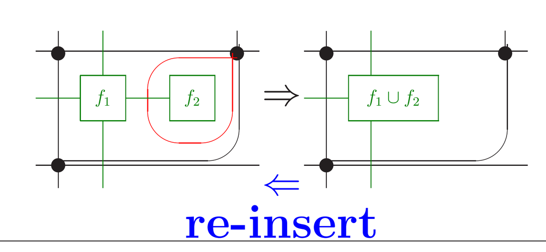


Dual Contrast Pyramid [3]

$I \leftrightarrow V, E \leftrightarrow \Gamma_4$	$I \leftrightarrow V', E' \leftrightarrow E$
$G(V,E)$, $\overline{G}(F, \overline{E})$	$G'(V', E', \overline{G}(F, \overline{E})$
contract $c(e) = 0$ ↓ remove $c((v,v)) = 0$	de-contract ↑ re-insert
$G_0(V_0, E_0)$, $\overline{G_0}(F_0, \overline{E_0})$	$G'_0(V'_0, E'_0, \overline{G_0}(F_0, \overline{E_0})$
mark critical points: ⊕, ⊖, ⊗	
contract $c(e) = 1$ ↓ remove $c((v,v)) = 1$	de-contract ↑ re-insert
$G_1(V_1, E_1)$, $\overline{G_1}(F_1, \overline{E_1})$	$G'_1(V'_1, E'_1, \overline{G_1}(F_1, \overline{E_1})$
⋮	⋮
$G_{t-1}(V_{t-1}, E_{t-1})$, $\overline{G_{t-1}}(F_{t-1}, \overline{E_{t-1}})$	$G'_{t-1}(V'_{t-1}, E'_{t-1}, \overline{G_{t-1}}(F_{t-1}, \overline{E_{t-1}})$
contract $c(e) = c_t$ ↓ remove $c((v,v)) = c_t$	de-contract ↑ re-insert
$G_t(V_t, E_t)$, $\overline{G_t}(F_t, \overline{E_t})$	

contract edge | remove self-loop $(v, v) \in E$





Properties:

Contraction preserves connectivity;	Removal of a self-loop (with $\deg(f_2) = 1$)
… removes 1 edge and 1 vertex;	… merges faces f_1 and f_2;
… preserves degrees of dual faces.	= contraction of dual edge of (v, v);
Self-loops cannot be contracted.	… decreases degree of vertex v
Critical vertices are preferred survivors.	and preserves ALL other vertices.
Non-contracted edges inherit original contrast.	

Equivalent Contraction Kernel $\mathrm{ECK}(v_\lambda) \subset E$, $G/ECK(v_\lambda) = v_\lambda \in V_\lambda$

Equivalent Connecting Path $\mathrm{ECP}(v_t, w_t) \subset E$, $(v_t, w_t) \in E_t$

All edges $e \in \mathrm{ECP}(v_t, w_t)$ have a contrast $c(e) \leq c_t$.

The process **dual graph contraction**[1] preserves the **topology**.

Total Order (TO)

$h_c(z)$ does NOT determine the order among the sets $E_z = \{e \in E | c(e) = z\}$ $\Longrightarrow$ needs a strict total order (TO) **rank** $\rho : V \mapsto [1, |V|]$

With **max-link**(TO) every vertex $w \in V \setminus R_{max}$ chooses the vertex $v \in \Gamma(w)$ with highest rank. R_{max} are local maxima of TO:

$$T_\oplus = \{(w, v) \in E | w \in V \setminus R_{max}, v = \arg\max_{u \in \Gamma(w)} \rho(u)\}.$$

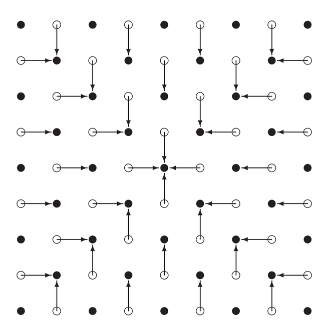
Column Major Order	Spiral TO	Max-Link(Spiral)
1 → 5 → 9 → 13	4 → 5 → 6 → 7	4 → 5 6 7
↓	↑ ↓	↓ ↓ ↓
2 → 6 → 10 → 14	3 14 → 15 8	3 → 14 → 15 ← 8
↓	↑ ↑ ↓ ↓	↓
3 → 7 → 11 → 15	2 13 16 9	2 → 13 → 16 ← 9
↓	↑ ↑ ↓	↑ ↑
4 → 8 → 12 → 16	1 12 ← 11 ← 10	1 → 12 11 ← 10
spanning tree $T_\oplus$	spiral links fill space	spanning tree $T_\oplus(\rho)$
root at the corner 16	… with path $1, \ldots, 16$	with central root 16
longest branch: 6	path length 15	longest branch: 4

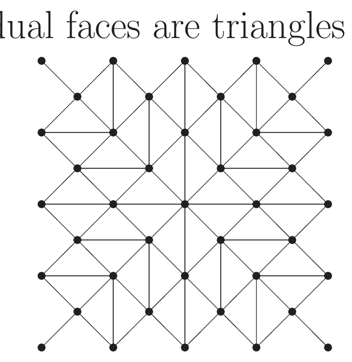
Contract a 9×9 Max-Link(Spiral)

- Graph $G(V, E)$ has 81 vertices V, 144 edges E, and 64 dual square faces F.
- spiral ranks: Odd (∘) are adjacent to even (•) ranks.
- Connected contraction kernels (CK): $\{\circ \longrightarrow \bullet\}$
- … are independent and can be contracted in parallel.

$G/CK_1 =$ $G_1(V_1, E_1)$ with 45 vertices

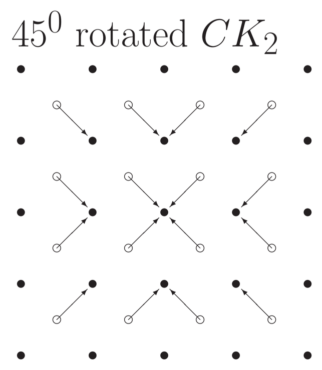
dual faces are triangles

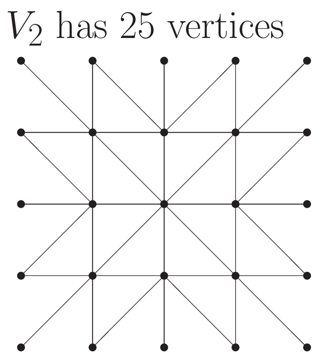




45^0 rotated CK_2

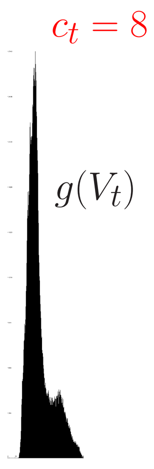
V_2 has 25 vertices



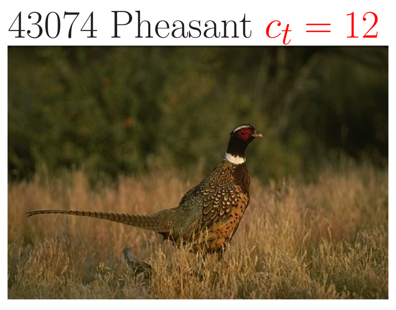


Orig. BERKELEY

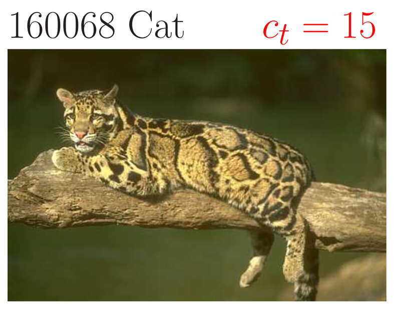
210088 Fish $c_t = 8$



43074 Pheasant $c_t = 12$

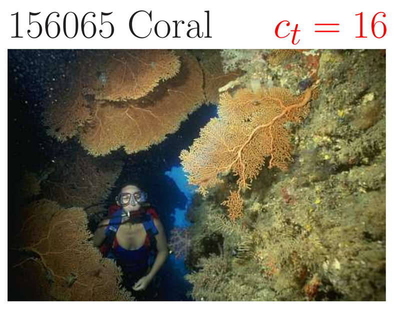


160068 Cat $c_t = 15$

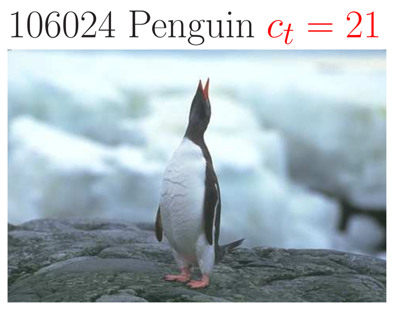


295087 Arch $c_t = 16$

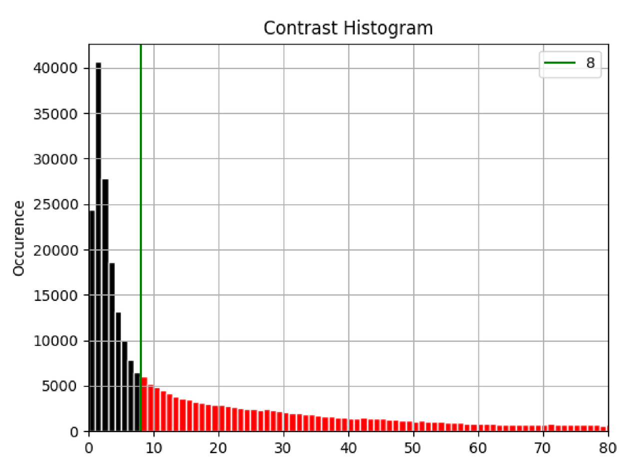
156065 Coral $c_t = 16$

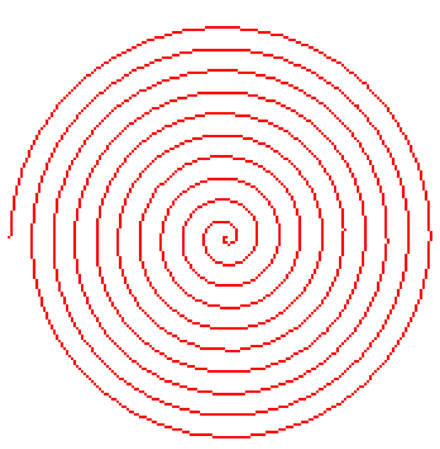


106024 Penguin $c_t = 21$



Contrast Histogram of Fish





Archimedean Spiral

16 different Spirals

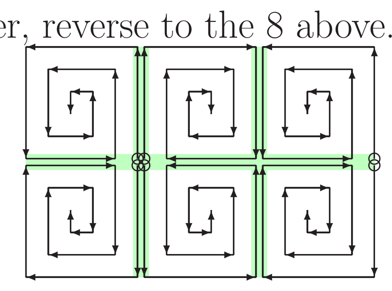
Spirals can start at the 4 corners of the image and end in the center.

2 orientations: clockwise - counter clockwise.

The spiral can start in the center, reverse to the 8 above.

Spirals could also be **tiled**:

… but borders must match!



Maximally contractible edges in the Berkeley DB [4]

Berkeley#	Picture	c_t	⊕, ⊖, ⊗	$\|V_t\|$
210088	Fish	8	18019	9264
43074	Pheasant	12	19788	4632
160068	Cat	15	20541	9264
156065	Coral	16	45915	12352
295087	Arch	16		12352
41069	Squirrel	25		9264

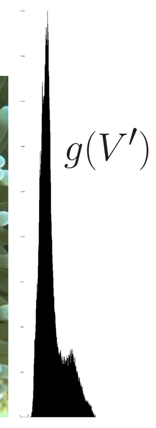
For $c_t < 20$:

the reconstruction is very similar to the original!

The mean deviation of reconstructed pixels $< c_t$ [2].

Reconstructions

210088 Fish



43074 Pheasant

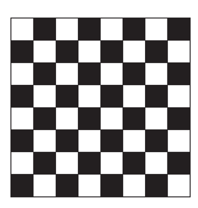
160068 Cat

295087 Arch

156065 Coral

106024 Penguin

Special Images

8×8 checkerboard	$m \times 256$ ramp
	
$h_c(0) = 0$	$h_c(0) = 256(m-1) < 256m - 1 : c_t = 0$
$h_c(255) = 112$	$h_c(1) = 255m$

Conclusion

- The dual contrast pyramid is the first HIGH-PASS pyramid.
- The top needs much less storage than the original and
- … provides very good reconstructions.
- CH provides TO, limits contractions (c_t) and
- provides a limit of the mean error [2].
- Spiral and Max-Link resolve ambiguous decisions.

Future Research

- tiling tiled spirals (see 16 spirals above)
- parallel algorithms
- other contrasts: color, texture (LBP ?), motion, …
- Discrete spirals/pyramids in 3D?
- **interest in collaborations? send email!**

References

[1] W. G. Kropatsch. Equivalent contraction kernels to build dual irregular pyramids. *Advances in Computer Science*, Advances in Computer Vision:pp. 99–107, 1997.

[2] W. G. Kropatsch. Lifting some Secrets about Contrast Pyramids. In S. Bougleux, L. Brun, V. Carletti, and B. Gaüzère, editors, *14th Intl. Workshop on Graph-Based Representations in Pattern Recognition*, volume 15727, pages 235–245. Springer LNCS, 2025.

[3] W. G. Kropatsch, M. Banaeyan, and R. Gonzalez-Diaz. Controlling Topology Preserving Graph Pyramids. In M. A.El-Yacoubi, N. Vincent, and C. Kurtz, editors, *Emerging Topics in Pattern Recognition and Artificial Intelligence*, volume 9 of *Language Processing, Pattern Recognition, and Intelligent Systems*, chapter 2, pages 27–76. World Scientific, 2024.

[4] D. R. Martin, C. Fowlkes, D. Tal, and J. Malik. A Database of Human Segmented Natural Images and its Application to Evaluating Segmentation Algorithms and Measuring Ecological Statistics. In *8th ICCV 2001*, volume 2, pages 416–423, 2001.

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