

Nonlinear Approximation of Spatiotemporal Data Using Diffusion Wavelets

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► Motivation

- Recent concept of **Diffusion Wavelets** (Coifman and Maggioni, 2006) allows construction of wavelet bases for functions defined on other than R^n , such as certain domains, manifolds and **graphs**
- In this work:** study the use of **classical wavelet algorithms**, lifted to a **graph based setting**, concentrating on an algorithm for nonlinear approximation of **2d+time image data**

► Introduction

- Diffusion Wavelets: particular instance of **spectral graph theory**
- In place of the usual dilation on R^n : **diffusion operator** on the data (given as a weighted graph) as **scaling tool**, dividing its spectrum (set of eigenvalues) into sets of different 'frequencies', obtaining a **multiresolution analysis** and an **ONB for functions on the graph**
- Diffusion Wavelet coefficients code **structural similarity** of the data encoded as a graph
 - Nonlinear approximation via thresholding on the coefficients just like in the classical wavelet algorithms (compression, denoising), obtaining a **structure-preserving** compression of the data

► Outlook

- Nonlinear approximation on a graph: can be seen as a **first step towards structural spatiotemporal wavelet segmentation**
- In order to obtain a **true segmentation**: first idea is to use a Hidden Markov Model on the coefficient tree, classifying the coefficients as 'large' or 'small' and following them across resolution levels
- Segmentation method we work on: could be used as a way of a **combined spatial and motion segmentation** of a whole image sequence
 - instance of 'offline' tracking method (similar to the *normalized cut* tracking, involving a true multiresolution analysis on the data)

► Algorithm

Input: image sequence (2d + time data)

Output: nonlinear approximation of image sequence

- Build a **weighted graph G from the image data**

Vertices: whole set of pixels or subset (e.g. using a downsampled version of the sequences, by filtering or by a feature point selection procedure)

Edges and weights: encode the local relations between vertices, (i.e. difference of intensities, distance in space, feature point properties, information from a motion prediction or a combination of the above)

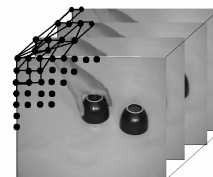
Function f on G: additional attributes on the vertices

- build a **diffusion wavelet basis** for functions on G
- compute **coefficients** of f with respect to this basis
- hard or soft **thresholding** on the coefficients
- reconstruction

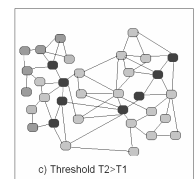
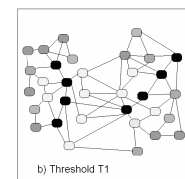
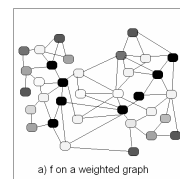
► Example

Visualization of the algorithm by an example

1. Build a weighted graph from an image sequence



2. Obtain an abstract graph



3. Thresholding of graph wavelet coefficients

Experimenting with different choices of vertices, weights and thresholds on real data will be part of our future work